
Coefficients of Restitution for Low and Moderate Speed Impacts with Non-Standard Impact Configurations

C. Brian Tanner, John F. Wiechel, Ron A. Bixel and Philip H. Cheng
S.E.A., Inc.

Dennis A. Guenther
The Ohio State Univ.

Michael P. Cassidy
Daimler Chrysler Corp.

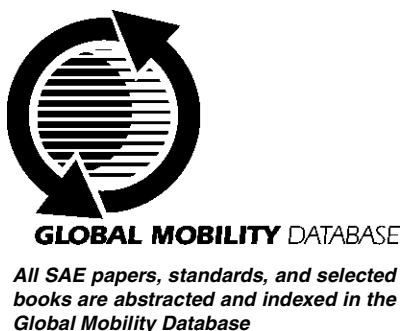
**Reprinted From: Accident Reconstruction-Crash Analysis
(SP-1572)**

The appearance of this ISSN code at the bottom of this page indicates SAE's consent that copies of the paper may be made for personal or internal use of specific clients. This consent is given on the condition, however, that the copier pay a \$7.00 per article copy fee through the Copyright Clearance Center, Inc. Operations Center, 222 Rosewood Drive, Danvers, MA 01923 for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law. This consent does not extend to other kinds of copying such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

SAE routinely stocks printed papers for a period of three years following date of publication. Direct your orders to SAE Customer Sales and Satisfaction Department.

Quantity reprint rates can be obtained from the Customer Sales and Satisfaction Department.

To request permission to reprint a technical paper or permission to use copyrighted SAE publications in other works, contact the SAE Publications Group.



No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.

ISSN 0148-7191

Copyright 2001 Society of Automotive Engineers, Inc.

Positions and opinions advanced in this paper are those of the author(s) and not necessarily those of SAE. The author is solely responsible for the content of the paper. A process is available by which discussions will be printed with the paper if it is published in SAE Transactions. For permission to publish this paper in full or in part, contact the SAE Publications Group.

Persons wishing to submit papers to be considered for presentation or publication through SAE should send the manuscript or a 300 word abstract of a proposed manuscript to: Secretary, Engineering Meetings Board, SAE.

Printed in USA

Coefficients of Restitution for Low and Moderate Speed Impacts with Non-Standard Impact Configurations

C. Brian Tanner, John F. Wiechel, Ron A. Bixel and Philip H. Cheng
S.E.A., Inc.

Dennis A. Guenther
The Ohio State Univ.

Michael P. Cassidy
Daimler Chrysler Corp.

Copyright © 2001 Society of Automotive Engineers, Inc.

ABSTRACT

There have been a number of papers written about the dynamic effects of low speed front to rear impacts between motor vehicles during the last several years. This has been an important issue in the field of accident analysis and reconstruction because of the frequency with which the accidents occur and the costs of injuries allegedly associated with them. Several of these papers have discussed the importance of the coefficient of restitution in the accelerations and speed changes that the vehicles undergo in such impacts. These discussions often include data showing the measured restitution for impacts involving various bumper types and closing speeds. However, in most of these studies, the impacts are controlled so that direct bumper to bumper impacts occur.

This paper will present the results of several rear impact tests with non-standard impact configurations. These configurations include several passenger car bumper to light truck and SUV trailer hitch impacts, and partial underride impacts to vehicles with rear mounted spare tires. The results of these tests will be compared with the findings from several tests with more standard impact configurations, and the coefficients of restitution for each of the impact scenarios will be discussed.

INTRODUCTION

When an aligned impact between a moving motor vehicle and a stopped motor vehicle occurs, the momentum transfer between the vehicles can result in three different outcomes for the post impact speeds. First, the vehicles can reach a common velocity following the impact. Second, the striking vehicle can come to a stop while the

struck vehicle rolls away. Finally, and most commonly, each of the vehicles moves following the accident with some finite separation speed.

Which of these three outcomes occurs depends on a property of the collision called the coefficient of restitution. The coefficient of restitution can be defined as the ratio of the separation velocity between the vehicles following the impact to the approach velocity of the vehicles just before the impact. Essentially, it is a measure of how much the vehicles "bounce" off of each other in the impact. Thus, in an impact where the vehicles reach a common velocity, the coefficient of restitution is zero since the separation velocity immediately after the impact is zero.

In order for energy to be conserved during the impact and for the second law of thermodynamics to be obeyed, the separation velocity of the vehicles following the impact can not be greater than the approach velocity of the vehicles before the impact. Thus, the maximum possible value for the coefficient of restitution is one. In an impact where the coefficient of restitution is one, the separation and approach velocities of the vehicles would be the same. In such a case, the post impact velocities of each of the vehicles depends on the masses of the vehicles. For example, if a moving vehicle strikes a stationary vehicle having the same mass and the coefficient of restitution for the impact is one, the post impact speed of the striking vehicle will be zero, while the post impact speed of the struck vehicle will be the same as the pre-impact speed of the striking vehicle. Conversely, if the striking vehicle hits an infinitely massive object or vehicle, and the coefficient of restitution is one, the post impact speed of the striking vehicle will be equal in magnitude but opposite in direction to the pre-impact speed of the vehicle, while the velocity of the massive object will remain zero.

The vast majority of impacts between vehicles have coefficients of restitution between zero and one. Generally, in higher speed vehicle impacts, the coefficient of restitution is small (0.1-0.2), and is usually ignored in determining vehicle impact speeds with acceptable accuracy. However, there can be exceptions to this generalization, and this will be discussed later in the paper. In low speed impacts, restitution between the vehicles is significant, and failure to account for it can result in relatively large errors in the determination of pre- and post impact speeds for the vehicles.

Researchers at MacInnis Engineering Associates published the results of some testing to determine the coefficients of restitution for some vehicle to vehicle impact configurations.¹⁻³ In general, they found that for low speed impact between vehicles equipped with bumpers mounted to piston style isolators, and both similar styled vehicles as well as rigid barriers, the coefficients of restitution ranged from about 0.2 to about 0.8. The largest values within this range typically occurred for barrier impact, and the restitution for a given impact configuration decreased with increasing impact speeds. It should be noted that in all of these impacts, care was taken to ensure that the vehicle contact was directed through the bumpers of the vehicles.

In another series of tests conducted by Szabo and Welcher², early model Ford Escorts were collided within a range of low speeds, and with bumpers both aligned and arranged so that underride would occur. They found coefficients of restitution ranging from about 0.1 to 0.4. Once again, the restitution generally decreased with increasing impact speed.

A third study conducted by MDE Engineers⁴ specifically examined the effect of bumper underride by the striking vehicles by comparing the results of rigid barrier impacts to underride impacts with a rigid bar for a number of vehicles. They observed coefficients of restitution ranging from about 0.2 to about 0.5.

METHODS

Although the data detailed above is useful, many modern vehicles have abandoned the use of bumpers with piston style energy absorbers and instead use structural foam, or molded plastic shapes enclosed by a urethane cover in their bumper designs. In addition, probably about half of the real world low speed impacts do not occur with straightforward bumper to bumper impacts, but involve some degree of misalignment, underride, or contact with non-standard vehicle components.

This paper describes the results of low and moderate speed impact tests between a variety of vehicles in a variety of configurations. These include a series of barrier impacts at several speeds between a 1993 Volvo 850 with a foam front bumper energy absorber, a series

of tests in which 1989 Chrysler LeBarons impact the rear of a 1988 Dodge ¾ ton 4 X 4 pickup with a class 3 trailer hitch, and two tests involving the impact of 1994 Dodge Grand Caravans into the rear of 1995 Suzuki Sidekicks with rear mounted spare tires.

VOLVO TO BARRIER IMPACTS - A total of three impacts were conducted in this configuration with impact speeds of about 9.26, 11.21, and 20.89 kilometers per hour. Figure 1 shows the vehicle and the impact configuration, while Figures 2 and 3 show an undamaged vehicle.



Figure 1 - 1993 Volvo 850/Barrier Impact Configuration



Figure 2 - Frontal View of the Undamaged Volvo

During each of the tests, tri-axial vehicle accelerations were measured by rigidly mounted accelerometers near the vehicle center of gravity. In addition, the impacts were recorded with both normal speed video and high speed film. Once the tests were completed, the acceleration data was digitally filtered to Channel Class 60 and the results were integrated to determine the vehicle speed change during the impact. The damage was photographically documented, and the coefficient of

restitution was determined by dividing the impact speed by the difference between the speed change and the impact speed. Figures 4-9 show the impact damage and the acceleration and speed change data for the tests.



Figure 3 - Side View of the Undamaged Volvo



Figure 6-Damage After the 20.89 kph Impact.



Figure 4 - Bumper Surface Damage after the 11.21 kph Impact.



Figure 5 - Side View after the 11.21 kph Impact.



Figure 7-Side View of the 20.89 kph Impact Damage.

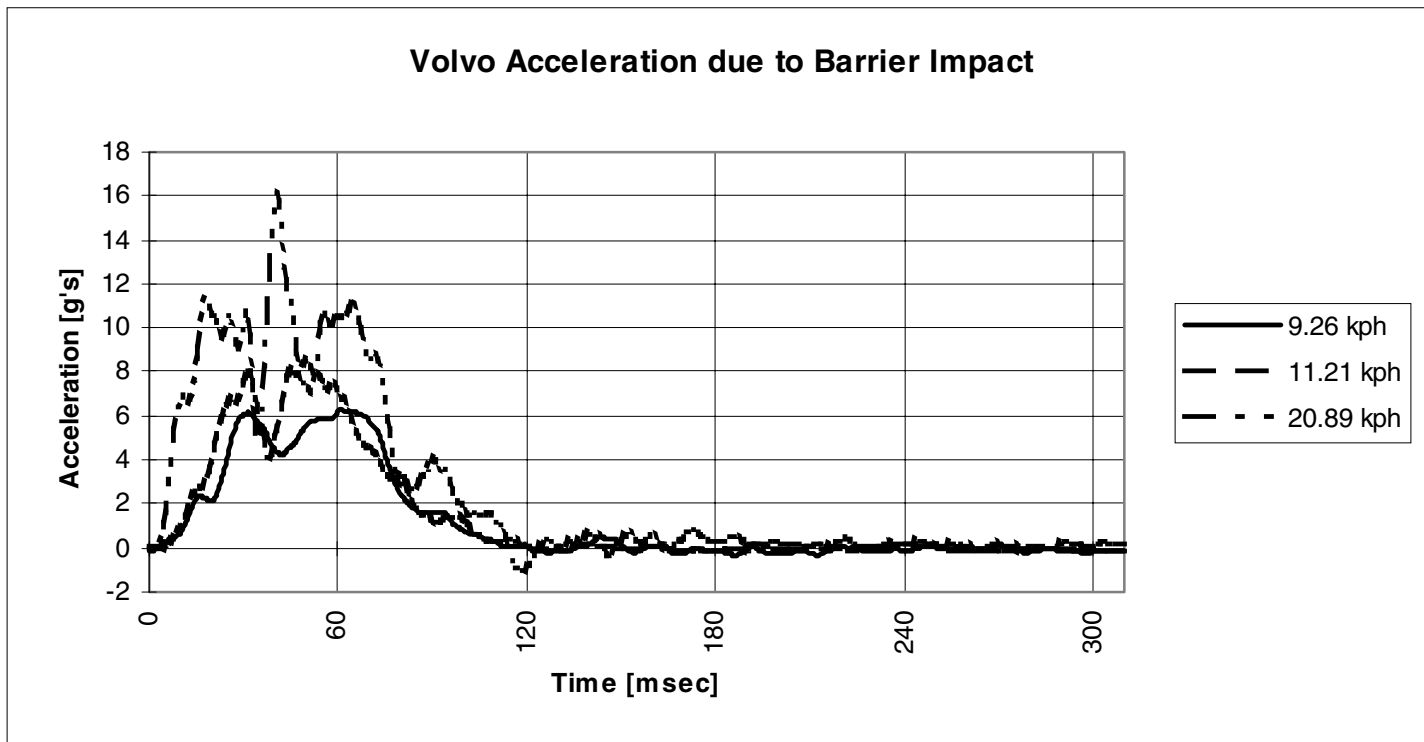


Figure 8

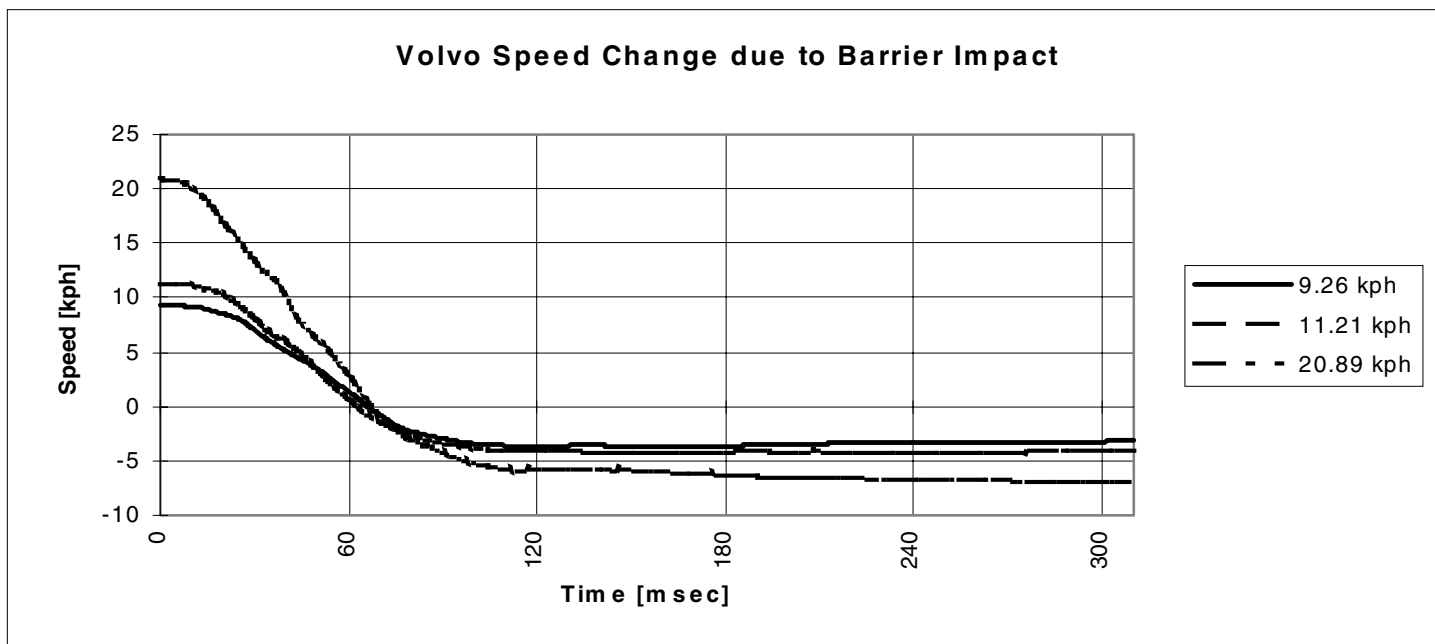


Figure 9

LEBARON TO 4 x 4 PICKUP WITH TRAILER HITCH IMPACTS - In this testing, a series of 4 impacts were conducted between the front of a 1989 Chrysler LeBaron and the rear of a 1988 4 X 4 Dodge $\frac{3}{4}$ ton pickup with a class 3 trailer hitch. A different LeBaron was used for each test, and the impact speeds were 11.29, 11.45, 17.74, and 19.35 kilometers per hour. In each case, the Dodge pickup was stopped prior to impact.

The alignment of the vehicles was controlled so that the impact occurred between the trailer hitch ball on the pickup and the center of the bumper on the LeBaron. In addition, the LeBarons were offset toward the right of the pickup by about 25 centimeters, and there was a slight left to right heading of the cars. Tri-axial accelerations were measured by rigidly mounted accelerometers near the center of gravity of the LeBaron's, and standard video and high speed film was shot of each of the impacts from several angles.

Figures 10 and 11 show the impact alignment of the vehicles for this test series, and Figure 12 shows the pre-impact, undamaged configuration for the Chrysler LeBarons. It should be noted that the design of the Chrysler LeBaron front bumper consists of a stamped metal bumper reinforcement bar behind a plastic cover. The bar attaches to the vehicle chassis through stamped metal energy absorbers shaped so that they will buckle at a certain level of load. The bumper on the Dodge pickup was stamped metal attached directly through stamped metal brackets to the vehicle frame. The trailer hitch consisted of a heavy steel receiver section that bolted directly to the pickup frame, with a heavy steel insert and ball that pinned into the receiver.



Figure 10- The LeBaron to pickup Impact Configuration.

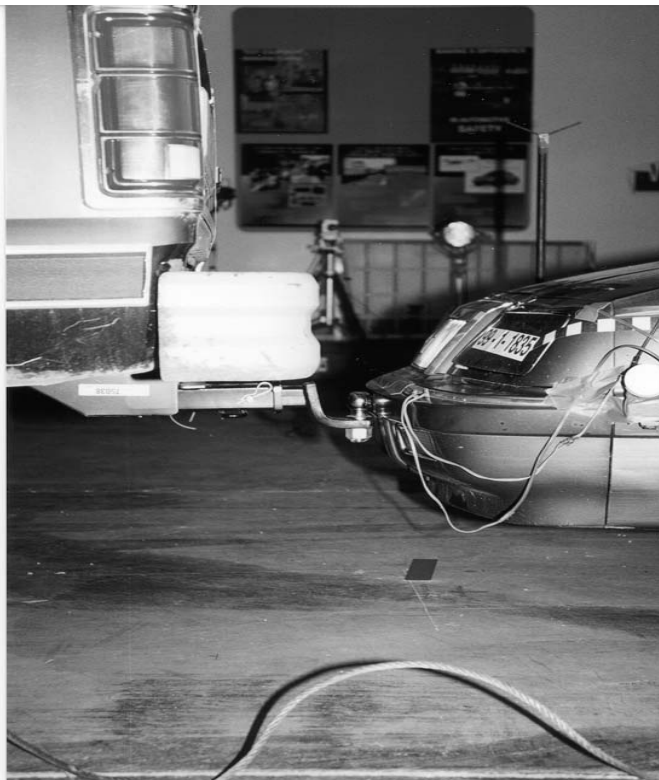


Figure 11 - LeBaron to Pickup Impact Alignment.

Except for the color, each of the LeBaron's in the impacts was essentially identical to the configuration shown in Figure 12. The pickup was not damaged in any of the tests and was reused each time.

Although care was taken to achieve the same impact alignment for each of the tests, the tests were performed on different days, and the rear of the truck had to be elevated for two of the tests to achieve the same bumper to trailer hitch alignment. Possibly as a result of this, and the geometry of the LeBaron bumper reinforcement bar, which had an opening cut into its front face, the pickup and LeBaron stuck together during each impact on the last day of testing. However, in all of the tests there was penetration of the LeBaron bumper cover by the trailer hitch ball, and examination of the high speed film indicated that even in the tests where the vehicles remained together, immediately following the maximum penetration of the trailer hitch into the bumper cover, the vehicles began to separate before they settled back together. Because the speed change of the pickup was determined by analysis of the high speed film shot at 500 frames/second, it was determined during the post impact vehicle separation phase before common velocity developed. Figures 13 to 20 show the external and internal damage to the LeBarons as a result of the impacts. Figures 21 and 22 show the impact accelerations and speed changes for the LeBarons.



Figure 12 - Undamaged LeBaron. Target shows Hitch Ball Impact Point.



Figure 13 - External Damage in the 11.29 kph Impact. Target indicates Hitch Ball Impact Position.



Figure 15 - External Damage in the 11.45 kph Impact.



Figure 14 - Reinforcement Bar Damage from the 11.29 kph Impact



Figure 16 - Reinforcement Bar Damage from the 11.45 kph Impact



Figure 17 - External Damage in the 17.74 kph Impact



Figure 19 - External Damage in the 19.35 kph Impact

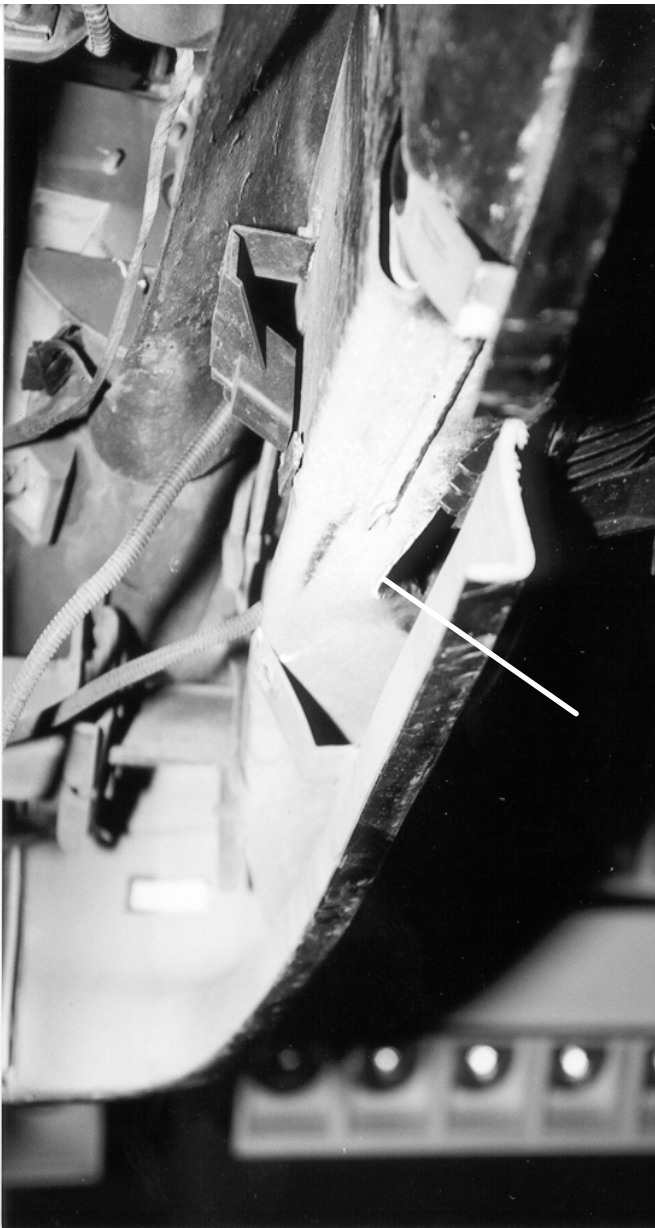


Figure 18 - Reinforcement Bar Damage in the 17.74 kph Impact.



Figure 20 - Reinforcement Bar Damage in the 19.35 kph Impact.

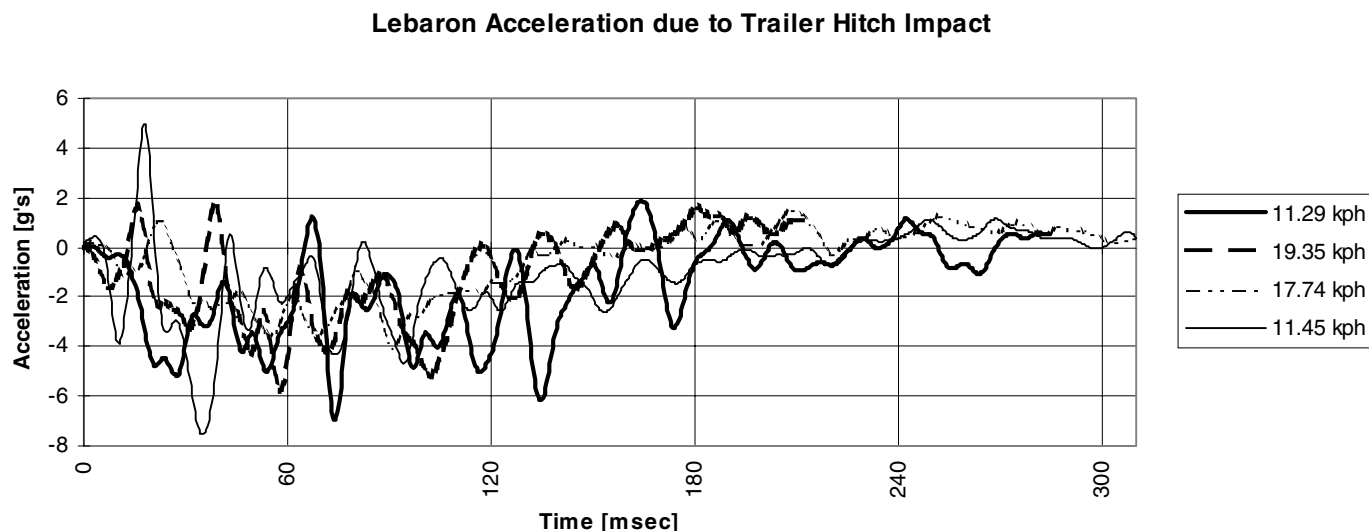


Figure 21

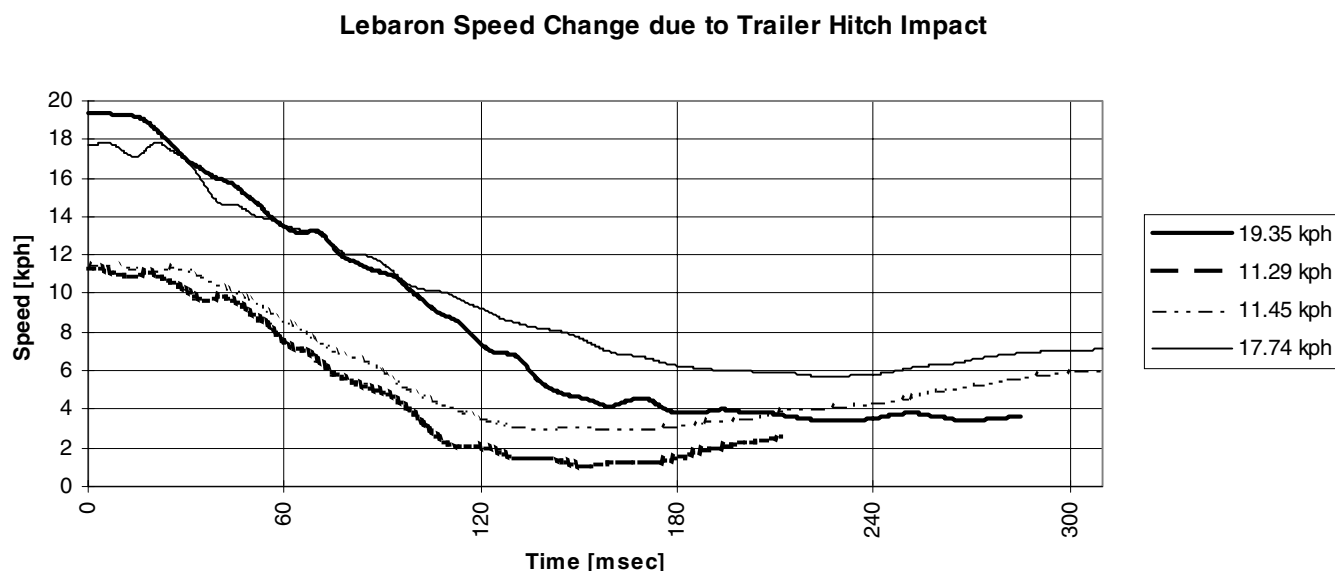


Figure 22

IMPACT OF 1994 DODGE GRAND CARAVANS TO 1995 SUZUKI SIDEKICK WITH REAR MOUNTED SPARE TIRE - For this test series, moderate impact speeds of 18.71 and 32.42 kilometers per hour were used. In addition, the front of the Grand Caravans were lowered to simulate a braked configuration. In this orientation, the upper half of the Grand Caravan front bumpers contacted the lower half of the Suzuki rear bumpers. Perhaps more importantly, at the impact speeds used in these tests, contact also occurred between upper grill, radiator support and hood edge of the Grand Caravan and the lower portion of the inflated, rear mounted spare tire on the Suzuki. Once again, in these tests, the Grand Caravans were offset to the right of the Suzukis nearly 30.5 centimeters. Neither the Grand Caravans or the Suzukis have any energy absorbers incorporated into their bumper designs.

As in the other test series, the Grand Caravan as the striking vehicle was instrumented with tri-axial accelerometers rigidly mounted near the vehicle center of gravity. In addition, for this test series, the Suzukis were also instrumented with tri-axial accelerometers near their centers of gravity. The tests were again documented with standard video and high speed film from multiple angles.

Figure 23 shows the impact configuration for the tests in this series, and Figures 24 and 25 show the front of one of the Grand Caravans and the rear of one of the Suzukis in their undamaged conditions before impact. Figures 26-29 show the vehicle acceleration and speed change data for the tests, and Figures 30-33 show the damage to the vehicles from the impacts.



Figure 23 - Impact Configuration of the Grand Caravan into the Suzuki Sidekick.



Figure 24 - Pre-impact geometry of the Grand Caravan



Figure 25 - Pre-impact Geometry of the Suzuki Sidekick

Grand Caravan Acceleration due to Suzuki Rear Spare Tire Impact

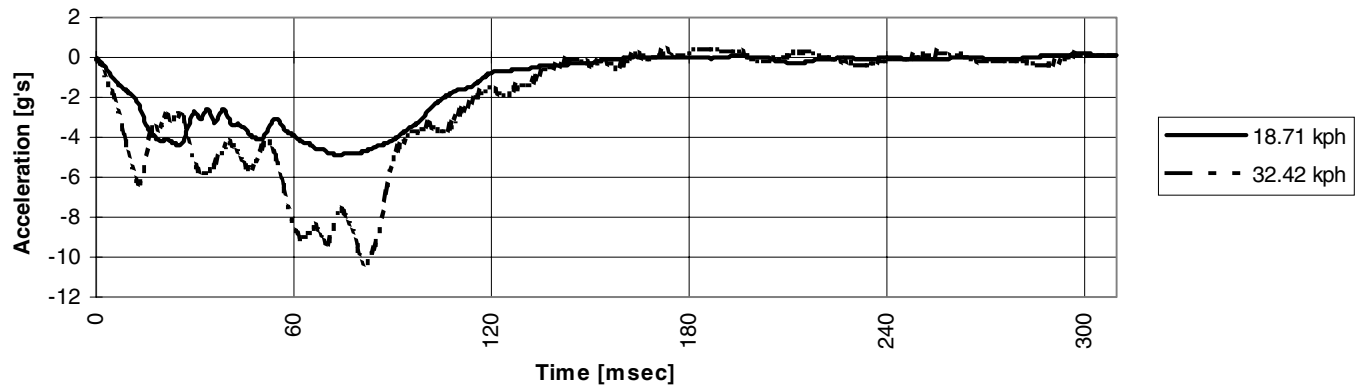


Figure 26

Suzuki Acceleration due to Grand Caravan Rear Spare Tire Impact

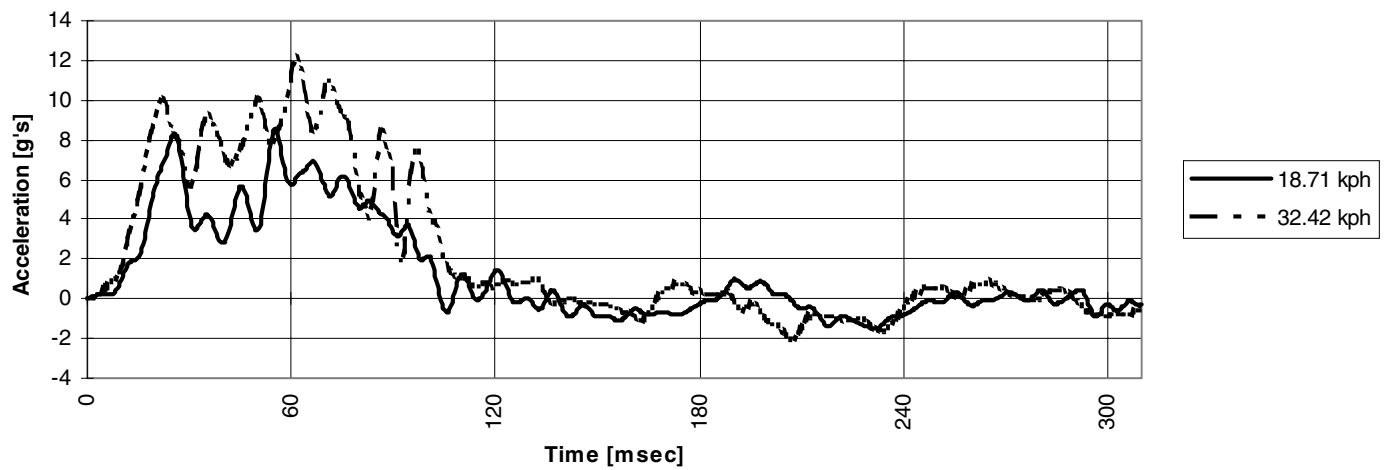


Figure 27

Grand Caravan Speed Change due to Suzuki Rear Spare Tire Impact

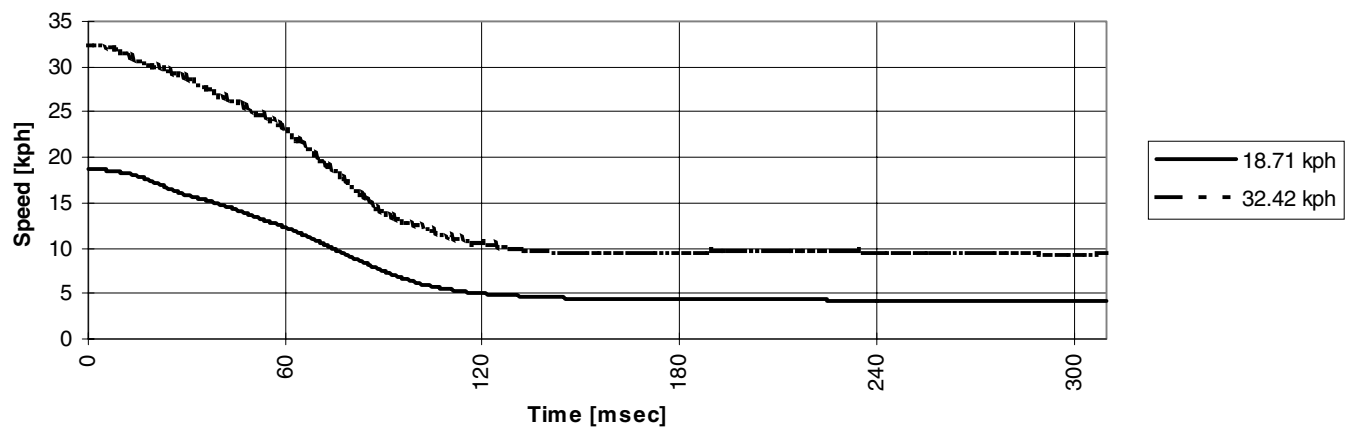


Figure 28

Suzuki Speed Change due to Grand Caravan Rear Spare Tire Impact

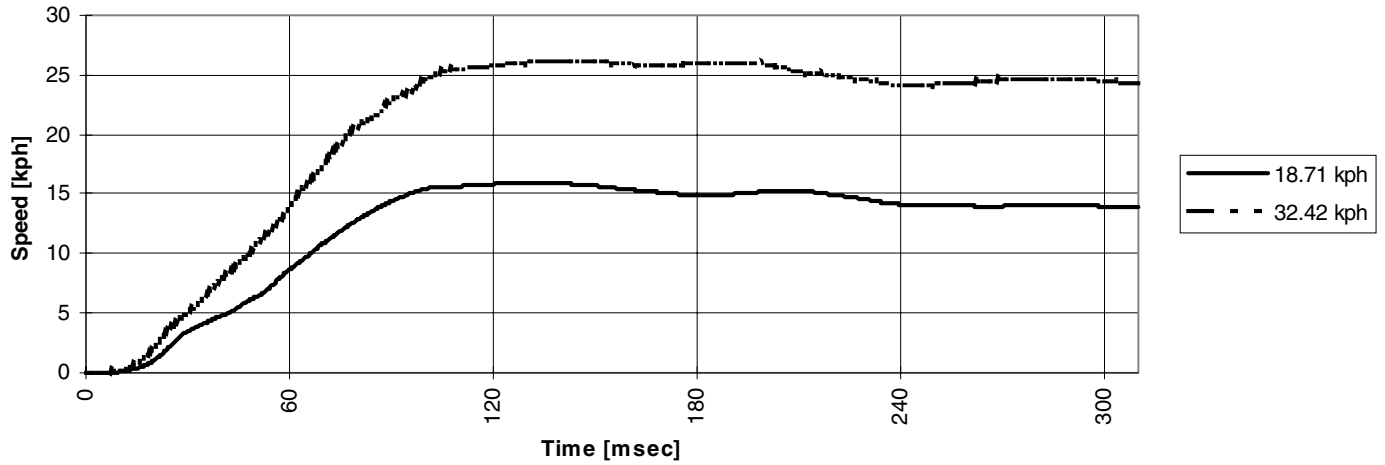


Figure 29



Figure 30 - Grand Caravan Damage in the 17.91 kph Impact



Figure 32 - Suzuki Damage in the 17.91 kph Impact



Figure 31 - Grand Caravan Damage in the 32.42 kph Impact



Figure 33 - Suzuki Damage in the 32.42 kph Impact

Table 1 - Summary of Test Results

Bullet Vehicle	Target Vehicle	Impact Speed [kph]	Bullet Vehicle ΔV [kph]	Target Vehicle ΔV [kph]	Restitution
1993 Volvo 850	Rigid Barrier	9.26	-12.9	0	.39
1993 Volvo 850	Rigid Barrier	11.21	-15.35	0	.37
1993 Volvo 850	Rigid Barrier	20.89	-26.82	0	.28
1989 Chrysler LeBaron	1988 Dodge ¾ ton 4 X 4 pickup	11.29	-10.06	4.81	.32
1989 Chrysler LeBaron	1988 Dodge ¾ ton 4 X 4 pickup	11.45	-8.45	4.77	.15
1989 Chrysler LeBaron	1988 Dodge ¾ ton 4 X 4 pickup	17.74	-12.05	6.94	.07
1989 Chrysler LeBaron	1988 Dodge ¾ ton 4 X 4 pickup	19.35	-15.16	8.37	.22
1994 Dodge Grand Caravan	1995 Suzuki Sidekick	18.71	-14.31	15.94	.62
1994 Dodge Grand Caravan	1995 Suzuki Sidekick	32.42	-23.05	26.19	.52

DISCUSSION - The results of the tests described in this paper are summarized in Table 1 above. As the table shows, for the Volvo to Barrier and Grand Caravan to Suzuki tests, the trend of decreasing coefficient of restitution with increasing impact speed identified for bumper to bumper impacts is also apparent here. The trend does not seem to apply to the LeBaron to pickup with trailer hitch impacts upon first examination, since one of the higher restitution results occurred at the highest impact speed. However, comparing the results from the tests at impact speeds of 11.29 and 11.45 kilometers per hour and the results from the tests at impact speeds of 17.74 and 19.35 kilometers per hour suggests that the reason for this deviation from the trend is related to a problem with test repeatability. In fact, if the four LeBaron to pickup impact tests were further subdivided into impacts where the vehicles separated following the impact, or stuck together following the impact, the same trend observed for the other impact configurations would also be valid here. That is, the restitution for the 19.35 kilometers per hour impact is significantly decreased from that for the 11.29 kilometer per hour impact, and the restitution for the 17.74 kilometer per hour impact is significantly reduced from that for the 11.45 kilometer per hour impact.

Another interesting feature of these tests is that the least amount of restitution occurred for the LeBaron to pickup impacts, while by far, the highest amount of restitution occurred for the Grand Caravan to Suzuki impacts, despite the fact that the highest impact speeds were utilized in the Grand Caravan to Suzuki impacts. The reason for this is most likely that the impact configuration between the Grand Caravan and Suzuki allowed the hood of the Grand Caravan to engage only the inflated portion of the Suzuki spare tire and wheel assembly. In addition, the impacts were not sufficient to cut or debond the Suzuki spare tire. Thus, the tire acted as an elastic

bumper, allowing the vehicles to bounce off of each other. This behavior would be very sensitive to the relative impact heights of the vehicles and might not be repeated for a slightly different impact alignment.

Finally, despite the differences in impact speed and configuration among the tests described in this paper, the overall range of impact duration was only about 120 to 180 milliseconds. The longest impact duration always occurred in the LeBaron to pickup impacts, which also generally had the lowest coefficients of restitution and the greatest amount of bumper structure plastic deformation. It is likely that the narrow contact area in this configuration for similar impact speeds allowed the interaction of the vehicles to occur over a greater travel distance, thus the impact pulse duration was also extended. A consequence of this is that for similar impact speeds above the damage threshold, a vehicle usually undergoes a lower average impact acceleration for the same speed change. In addition, even though two impacts might involve objects having the same masses and speeds, a narrower contact area can result in a lower overall speed change because of a reduced coefficient of restitution.

CONCLUSION

This paper describes the results of several tests involving a variety of vehicles in a range of non-typical impact configurations. Based on the test results, several conclusions are possible:

1. The coefficient of restitution for the Volvo 850, which has a full foam energy absorber in its front bumper, ranges from about 0.2 to about 0.4 over the range of barrier impact speeds of about 9.5 to 21 kilometers per hour.

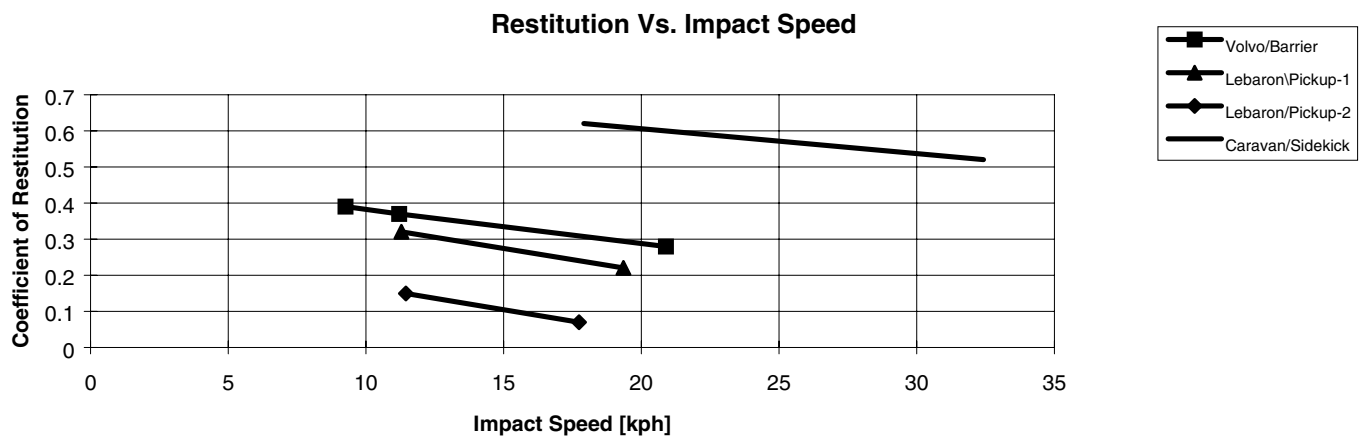


Figure 34

1. Even in non-typical impact configurations at low to moderate speeds, the coefficient of restitution for the impact tends to decrease with increasing impact speed. This is shown in Figure 34.
2. Impacts involving narrow contact areas generally result in more damage, longer impact duration, lower restitution, and smaller speed change and average acceleration than impacts at similar speeds with broader contact areas.
3. In certain circumstances, such as the Grand Caravan to Suzuki impacts, high restitution can result, even for moderate impact speeds, because of the particularly elastic nature of some component involved in the impact.
4. Siegmund, G.P., King, D.J., and Montgomery, D.T., Using Barrier Impact Data to Determine Speed Change in Aligned, Low-Speed Vehicle-to-Vehicle Collisions, S.A.E. Paper No. 960887, 1996.
5. Goodwin, V., Martin, D., Sackett, R, Schaefer, D.O., and Tencer, A., Vehicle and Occupant Response in Low Speed Car to Barrier Override Impacts, S.A.E. Paper No. 1999-01-0442, 1999.

While the results of the tests described in this paper provide some data that could serve as a guideline for the reconstruction of other accidents involving non-typical impact configurations, extension of the results should only be done with great care.

REFERENCES

1. Szabo, T.J. and Welcher, J., Dynamics of Low Speed Crash Tests with Energy Absorbing Bumpers, S.A.E. Paper No. 921573, 1992.
2. Siegmund, G.P., Bailey, M.N., and King, D.J., Characteristics of Specific Automobile Bumpers in Low-Velocity Impacts, S.A.E. Paper No. 940916, 1992.
3. Bailey, M.N., Wong, B.C., and Lawrence, J.M., Data and Methods for Estimating the Severity of Minor Impacts, S.A.E. Paper No. 950352, 1995.