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ABSTRACT

The restitution or rebound that occurs as the final phase of a vehicle-to-vehicle collision is quantified by the coefficient of restitution, which is the ratio of the closing velocity to the post-impact separating velocity of the two colliding vehicles. The coefficient of restitution of medium and high velocity collisions is low, (approximately 0.1) since these collisions are quite inelastic, whereas collisions at extremely low velocities are relatively elastic with the coefficient of restitution theoretically approaching 1.0. However, the actual collision restitution magnitude in the low velocity range has not been adequately established. A series of vehicle-to-vehicle and vehicle-to-barrier collisions resulting in velocity changes in the 2 to 5 miles per hour range was conducted in which vehicles with various bumper configurations (factory standard equipment) were utilized to study the coefficient of restitution at low closing velocities. Data from each vehicle-to-vehicle collision in which a stationary vehicle was struck from the rear by another vehicle were recorded with high-speed photography and vehicle-mounted accelerometers. Coefficients of restitution measured from this series of collisions were clustered in the 0.2 to 0.4 range which is

significantly lower than previous extrapolations and testing have indicated. A qualitative analysis of the dynamics responsible for this lower than expected restitutive response is described. An analytical method to determine vehicle-to-barrier derived restitutive values is also presented.

INTRODUCTION

The potential for injury to occupants in vehicle-to-vehicle collisions is proportionally to related to the velocity change that the vehicles undergo during a collision interaction. At high closing velocities, in collinear vehicle-to-vehicle collisions, the impact is relatively inelastic and vehicle rebound is modest. However, at low closing velocities, the collision-related vehicle deformation is diminished and may be limited to dynamic deformation. The structures deforming during the collision event will then store energy which will produce a rebound as the deformed structures return elastically toward their pre-impact configuration. When the total collision energy is low, the elastic rebound energy has proportional significance in its contribution to the velocity change as compared to its very diminished contribution in higher energy collisions.

BACKGROUND

The relative elasticity of a collision is identified by the coefficient of restitution which is the ratio of the post-impact separating velocity of two colliding objects ($V_1 - V_2$) to their closing velocity. The coefficient of restitution (ϵ) equals 1.0 for a purely elastic collision and 0.0 for a purely plastic collision (no rebound).

The identification of coefficients of restitution in vehicle-to-vehicle collisions is impractical since each vehicle-to-vehicle combination has its unique restitutive response. However, vehicle-to-barrier coefficients of restitution can be measured for specific vehicles, and representative plots of closing velocity to the coefficient of restitution can be established. The coefficient of restitution applicable to a collision between two vehicles for which the vehicle-to-barrier coefficients of restitution are known may then be predicted. The interested reader may refer to Appendix A for the derivation of the relationship between vehicle-to-barrier and vehicle-to-vehicle coefficients of restitution.

It would appear that those vehicular structures which constitute the plastic-elastic elements contributing to the restitution following a low velocity collinear impact would be limited in most cases to the bumper assemblies. It would also seem reasonable that a coefficient of restitution versus closing velocity plot, rather than representing a linear function, may contain discontinuities which occur at closing velocities at which various structures comprising elements of a plastic-elastic restitution mechanism undergo structural failure. For example, when the relatively stiff bumper assembly undergoes structural failure and is no longer able to react elastically, the relatively pliable structures behind the bumper, i.e. fenders, grill, hood, and trunk lid, undergo deformation which may approximate a response which is more plastic than elastic. The stiffer frame members would then be encountered as the deformation continues, resulting in an increased elastic response, until these structures would undergo structural failure and would be no longer able to store potential mechanical energy to produce an elastic rebound. Consequently, the coefficient of restitution would be expected to vary with respect to closing velocity with discontinuities in the curve at these respective failure points.

Other phenomena which contribute to the non-linearity of the coefficient of restitution, with respect to closing velocity, include the shear, moment, and tire forces which proportionately play a greater role as closing velocity diminishes.

Collins[1] described the coefficient of restitution in vehicle-to-vehicle collisions as a function of the closing velocity and noted that it varies from essentially zero* at velocity changes over 25 m.p.h. to nearly 1.0 at velocity changes approaching zero. By linear interpolation he predicted for collisions in which the velocity was low, i.e. 5 to 10 m.p.h., the coefficient of restitution would be in the range of 0.8 to 0.9. In contrast, Emori and Horiguchi[3] suggested the value of the coefficient of restitution at velocities of 2 to 3 m.p.h. would be in the range of 0.5 to 0.6. How-

* In contrast NHTSA sponsored barrier crash tests at 35 m.p.h. indicate impact restitution at these higher speeds is in the 0.1 to 0.15 range[2]

ever, their data were from collision tests into a barrier using modified bumper assemblies.

One objective of the collision tests reported below was to estimate the coefficient of restitution resulting from vehicle-to-vehicle collisions at the low end of the closure velocity range.

MATERIALS AND METHODS

Facilities and instrumentation for performing impact tests and data acquisition were provided by Southwest Research Institute of San Antonio, Texas. Test management and protocol were also provided by Biodynamic Research Corporation, also of San Antonio, Texas.

TEST VEHICLES - Vehicle-to-vehicle collisions were conducted using four motor vehicles with each vehicle utilized as the striking and struck vehicle in the various tests. The vehicles were:

1986 Dodge 600 Convertible - 2740 lbs.

1984 Buick Regal Limited Coupe - 3240 lbs.

1984 GMC C-1500 Pickup Truck - 3018 lbs.

1984 Ford E-150 Club Wagon Van - 4040 lbs.

Vehicles selected were without evidence of collision structural damage, in roadworthy condition, and free of non-standard equipment or modification. The struck vehicles were stationary with transmission in neutral and the brakes off. The impacts were between the front end of the striking vehicle and the rear end of the struck vehicle. Vehicle-to-vehicle impacts, in all but one case, were conducted such that direct bumper-to-bumper contact existed with a minimum of vertical height difference between the bumpers. Standard equipment bumpers and energy absorbers were installed to replace impact damaged equipment as needed for subsequent impact tests. Energy absorber mechanisms (standard equipment) constituted the bumper suspension units for the convertible and the coupe. All of these absorbers were of hydraulic design except for a set of replacement deformation absorbers on the Dodge. The pickup truck and van bumpers were bolted directly to the frame, per design, with no energy absorber system in use. Doors were removed from all vehicles to facilitate photographic coverage of occupant kinematic motion, another objective of the tests. The upper portion of the left B-pillar of the van was also removed for the same reason.

TEST TRACK - The collision tests were conducted on a level paved track. A ramp was constructed and calibrated to impart approximate impact velocities to the striking vehicles. Vehicles were positioned on the ramp under their own power and calibration runs were then performed to determine repeatable impact zone velocities.

The vehicles were driven by human volunteers. Biodynamic data relating to these human subjects' responses will be reported in future papers. Several vehicle-to-barrier collisions were conducted. The barrier was a 23,000 pound flat-faced concrete block, approximately eight feet wide, six feet high, and ten feet thick, which rested unattached on the paved track.

INSTRUMENTATION - A triaxial accelerometer array of LSCB-10 accelerometers was affixed close to the frame or unibody structure close to the vehicle center of gravity. Linear displacement transducers were placed at the front bumper of the striking vehicle to measure bumper displacement, relative to the vehicle frame. Vehicle impact speed was determined by a pressure switch speed gate in the impact zone. Time of impact was determined by a pressure switch mounted on the front bumper of the striking vehicle, which activated a strobe light for photographic correlation and produced a time signal in the data acquisition system. Accelerometer and impact velocity data were recorded on a PAC-5800 data acquisition system.

PHOTOGRAPHIC EQUIPMENT - Panning video cameras at 30 frames per second were used to obtain general photographic coverage of the test. Cameras positioned off-track on both sides at the impact zone were Redlake LoCam Model #51 16 millimeter movie cameras which operated at a speed of 500 frames per second. Each was equipped with a LED timing light generator set at 100 hertz. A high speed video camera was used to corroborate vehicle speeds and displacements.

Table 1 - Collision Test Series					
Test	Striking Vehicle	Struck Vehicle	Closing Velocity	Struck Vehicle Delta V	Coefficient of Restitution
1	Dodge Convertible	Van	3.8	2.2	0.36
2	Pickup	Van	6.2	4.0	0.25
2R	Pickup	Van	7.2	4.1	0.29
3	Van	Pickup	3.4	1.9	0.15
4	Van	Pickup	6.8	4.1	0.22
4R	Van	Pickup	7.2	4.4	0.26
5*	Coupe	Convertible			
6	Coupe	Convertible	7.6	5.0	0.30
7	Pickup	Coupe	3.6	2.4	0.23
8	Convertible	Coupe	8.5	4.9	0.25
9	Convertible	Barrier	4.7		0.26
10	Coupe	Barrier	2.3		0.37
11	Coupe	Barrier	4.9		0.29
12	Convertible	Barrier	1.0		0.86
13	Coupe	Barrier	4.7		0.33
13R	Coupe	Barrier	7.1		0.34

* aborted test

RESULTS

Table 1 shows the various vehicle-to-vehicle and vehicle-to-barrier impact tests performed in this series.

Figure 1 displays the estimated coefficients of restitution versus closing velocity measured in the series of nine vehicle-to-vehicle collisions. Closing velocity and impact velocity of the striking vehicle were identical, since the struck vehicle was stationary in all tests.

Figure 2 shows the coefficients of restitution estimated in the six vehicle-to-barrier tests.

Figure 3 shows a plot of vehicle velocity in test #8, a collision between the Dodge convertible and the Buick Regal sedan in which the Buick's Delta-V was 4.9 m.p.h. The reference point on each of the vehicles was taken at the base of the B-pillar which was assumed to move essentially with the center of gravity.

DISCUSSION

The coefficient of restitution resulting from the series of low closing velocity vehicle-to-vehicle impacts are of a significantly lower magnitude than that predicted by previous analyses[1] and tests[3]. A general progression toward greater restitution is suggested by the data as closing velocity diminishes, as would have been expected. However, impact restitution in the closing velocity range of 2.5 miles per hour remains for this series below 0.4, indicating a sharp rise in the curve would occur as the closing velocity approaches zero. The average coefficient of restitution for the nine vehicle-to-vehicle collision was approximately 0.25 in the velocity range of 2.5 - 5.0 m.p.h. The vehicle-to-barrier collision resulted in somewhat higher coefficients (0.3 to 0.4) for the same velocity range. The single test for the very low closing velocity (1.0 m.p.h.) produced a coefficient of restitution of 0.86, indicating the expected trend toward a nearly elastic response as the closing velocity approaches 0.

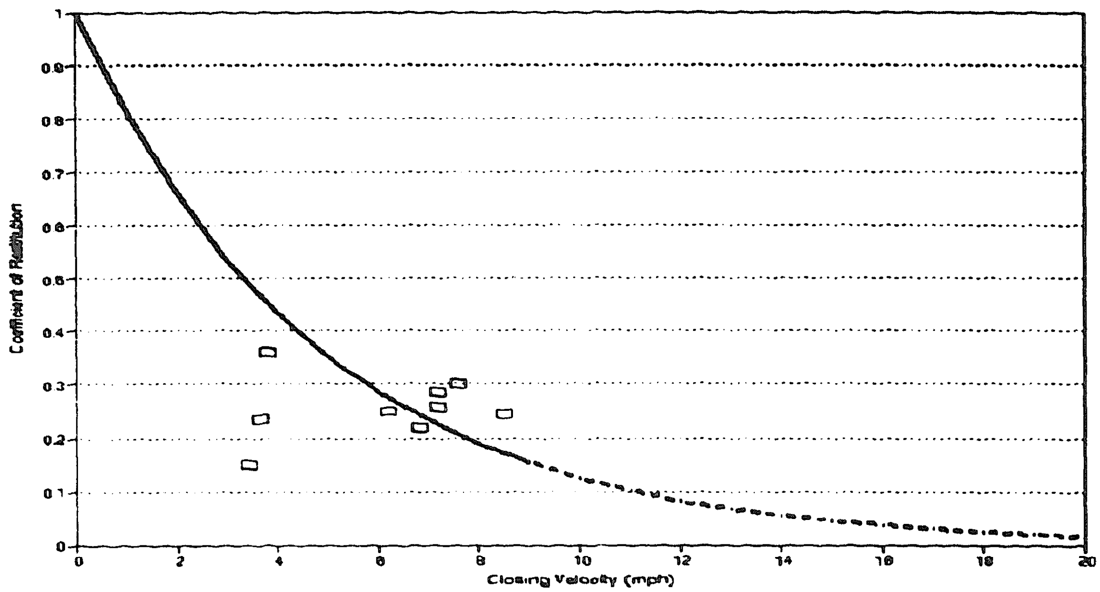

Figure 1 - Vehicle-to-Vehicle Data
Coefficient of Restitution Test 1-8

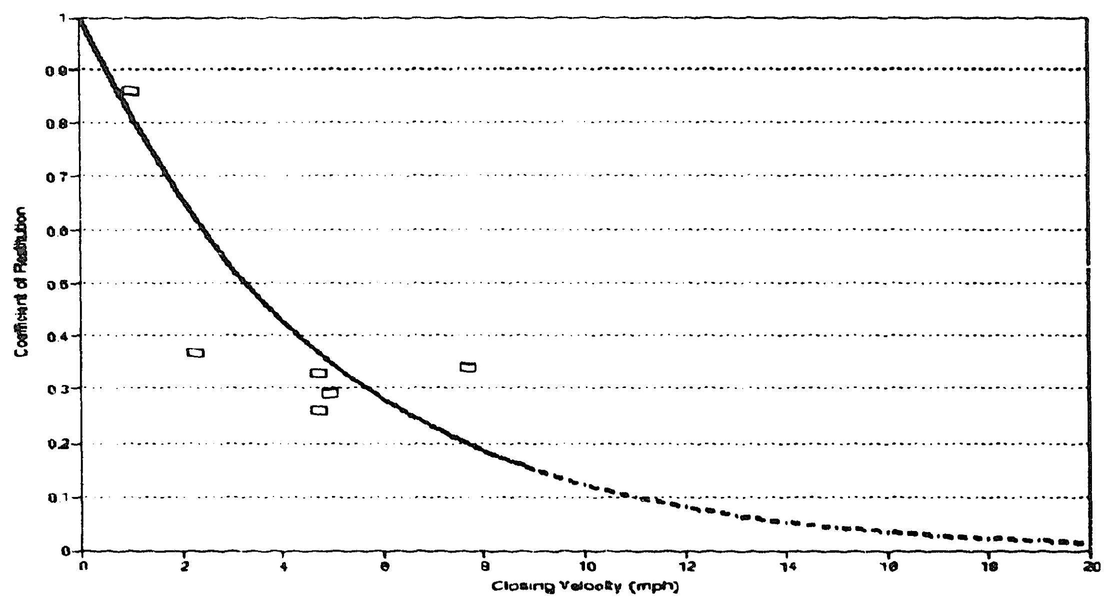

Figure 2 - Vehicle-to-Barrier Data
Coefficient of Restitution Test 9-13R

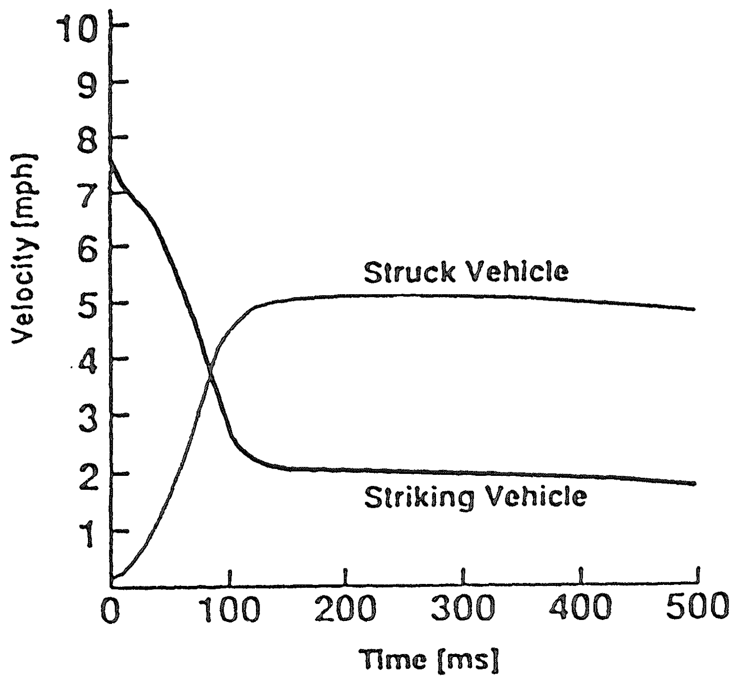

Figure 3 - Test 8 Velocity Plot

It is noteworthy that the greater magnitude coefficient of restitution expected in the pickup truck-van impacts, due to the absence of energy absorbers in their bumper assemblies, was not seen. This would suggest that, although the bumpers without plastic-elastic energy absorbers possess a greater capability for storing energy for an elastic response, other factors attenuate that response. Similarly, the less than expected restitution at low velocity closure rates may also be explained by these same factors. Although it is an obvious inaccuracy, it has been convenient to think of vehicles colliding at low velocity as being rigid bodies, with the areas of contact deforming in the collision without affecting the structural relationships in the dynamically "undeformed" remainder of the vehicle. In fact, significant relative movements of structural elements throughout the vehicle occur, each returning essentially to its pre-impact position, relative to the overall structure with frictional energy "loss" occurring with each movement. The rate at which these elements return to their pre-impact structural state may be significantly less than the rate of structural shifting during the "compressive" phase of the collision, resulting in a diminished elastic response quite different from that expected with rigid body impacts. In short, the vehicle response to a low velocity impact with another vehicle may more closely approximate a non-linear, plastic interaction than an elastic impact.

Other energy "sinks" not related to permanent deformation also exist. Wheel and transaxle assemblies, which represent a significant portion of the total vehicle mass, interact with the vehicle frame through the suspension system, and during the impact sequence, their movement with respect to the vehicle frame is dependent on the plastic-elastic characteristics of that system. Tire forces are similarly transmitted to the frame and the vehicle responds as a sprung mass. High speed movie film of the collisions in this series of tests shows the action of these phenomena. These and other impact vibrations of the components of the vehicle structure, depending on their frequency response, may contribute out of phase and in misdirection to the overall restitutive response of the vehicle.

The foregoing factors are generally reductive to the magnitude of the coefficient of restitution. Their independent effects on restitution would be difficult to identify quantitatively and would differ by vehicle and load. Their cumulative effect, however, can, for practical purposes, be quantified by the coefficient of restitution.

In our series of tests we measured the restitutive responses to the Dodge and Buick collisions into the barrier at closing velocities of 4.7 and 4.9 miles per hour respectively and the two collisions between these two vehicles in which the velocity changes of the struck vehicles were 4.9 and 5.0 miles per hour. The coefficients of restitution were:

ϵ_A = 0.26 (Dodge-barrier)

ϵ_B = 0.29 (Buick-barrier)

ϵ_{AB} = 0.25 and 0.30 (Dodge-Buick)

The calculated ϵ_{AB} per Eq.[10] in the Appendix was 0.274 which is consistent with the experimentally determined results.

In order to further define the coefficients of restitution to be expected over a spectrum of closing velocities, additional testing at intermediate (5 to 15 m.p.h.) and very low (0.5 to 2.0 m.p.h.) closing velocities is needed. These data would be useful in the more accurate determination of vehicular velocity changes resulting from low energy level collisions and in the further experimental confirmation of the proposed method of obtaining vehicle-to-vehicle collision coefficients of restitution.

REFERENCES

1. Collins, James C. Ph.D., Accident Reconstruction. Springfield, Illinois, Charles C. Thomas, Publisher Bannerstone House, 1979.

2. U.S. Department of Transportation National Highway Traffic Safety Administration, Report Nos. CAL-86-N-02, 212-CAL-84-150, 301-CAL-84-150, CAL-90-N10, 212-DYS-82-008, 219-DYS-82-008, 301-DYS-82-016, 212-DYS-82-012, 219-DYS-82-012, and 301-DYS-82-025.

3. Emori, Richard I. and Horiguchi, Junji, Whiplash in Low Speed Vehicle Collisions, Faculty of Engineering Seikei University, Musashino, Tokyo.

4. Romily, D.P., Ph.D., Low Speed Rear Impacts And The Elastic Properties of Automobiles, Proceedings 12th International Conference of Experimental Safety Vehicles 1989, Gothenburg, Sweden.

Appendix A

The rebound characteristics of a vehicle in a low velocity collinear collision wiht another vehicle in which the residual deformation damage is minor are essentially the same as the rebound characteristics of that vehicle in a collision with a barrier (or other vehicles or objects) in which the same deformation is produced and the duration of collision interaction is similar. (Collision interaction time in collinear, low velocity, vehicle-to-vehicle collisions would not be expected to vary significantly). The ratio of energy used in vehicle deformation to the total energy available for deformation in an impact is proportional to the square of the coefficient of restitution. Therefore, the coefficients of restitution measured in a collision between Vehicle A and a barrier (ϵ_A) and, similiarly Vehicle B and a barrier (ϵ_B), can be related to the coefficient of restitution of a collision between Vehicle A and Vehicle B (ϵ_{AB}) at selected impact velocities which would produce comparable residual deformation.

For a collision of a vehicle into a barrier:

where, M_A = mass of Vehicle A
V_A = pre-impact velocity of Vehicle A
V_A' = post-impact velocity of Vehicle A
E_{dA} = energy used in deformation of Vehicle A
E_{dB} = energy used in deformation of Vehicle B
E_C = total energy used in deformation

$$\frac{1}{2}M_A V_A^2 = E_{dA} + \frac{1}{2}M_A V'^2_A$$

and

$$\epsilon_A = \frac{V'_A}{V_A}$$

Therefore

$$E_{dA} = \frac{1}{2}M_A V_A^2 \left[1 = \epsilon_A^2\right] \qquad [1]$$

and, similarly,

$$E_{dB} = \frac{1}{2}M_B V_B^2 \left[1 = \epsilon_B^2\right] \qquad [2]$$

Using the result of Collins (Ref. 1), pp. 150-151

$$E_C = (1 - \epsilon^2)\frac{1}{2}\left[\frac{M_1 M_2}{M_1 + M_2}\right](V_1 - V_2)^2 \qquad [3]$$

where M_1 and M_2 are the respective masses and V_1 and V_2 are the respective pre-impact velocities of colliding vehicles 1 and 2 and ϵ is the coefficient of restitution of that collision.

Assume,

$$E_C = E_{dA} + E_{dB} \tag{4}$$

Expand [4] and rearrange to yield,

$$\epsilon = \left[1 + \frac{\left[M_A V_A^2(\epsilon_A^2 - 1) + M_B V_B^2(\epsilon_B^2 - 1)\right]\left[M_1 + M_2\right]}{M_1 M_2 (V_1 - V_2)^2}\right]^{\frac{1}{2}} \tag{5}$$

Now, consider a model in which two vehicles are closing at a velocity of (V_1 - V_2) and a movable barrier traveling at the velocity of the center mass of the two vehicles (V_{cm}) is interposed between them at their point of impact. The vehicles' respective impacts with the barrier produce damage essentially identical with the damage which would occur with the barrier absent for a low velocity collinear collision. The velocities of the vehicles relative to the barrier would be:

$$V_A = V_1 - V_{cm} \tag{6}$$

$$V_B = V_2 - V_{cm} \tag{7}$$

where,

$$V_{cm} = \frac{M_1 V_1 + M_2 V_2}{M_1 + M_2}$$

Now, let $M_1 = M_A$ and $M_2 = M_B$ and $\epsilon = \epsilon_{AB}$. Then,

$$V_A = \frac{M_B(V_1 - V_2)}{M_A + M_B} \tag{8}$$

and

$$V_B = \frac{-M_A(V_1 - V_2)}{M_A + M_B} \tag{9}$$

Combining Equations [5], [8] and [9] and simplifying:

$$\epsilon_{AB} = \left[1 + \frac{M_B(\epsilon_A^2 - 1) + M_A(\epsilon_B^2 - 1)}{M_A + M_B}\right]^{\frac{1}{2}} \tag{10}$$

ϵ_A and ϵ_B may be obtained from vehicle to barrier collision measurements at selected collision velocities near V_A and V_B. Estimation of the pre-impact velocities, V_1 and V_2, of two colliding vehicles, A and B, permits the selection of V_A and V_B by Equations [8] and [9]. The vehicle-to-vehicle collision coefficient of restitution, ϵ_{AB}, may then be calculated by Equation [10]. This value, ϵ_{AB}, relates to impact produced vehicle velocity changes, ΔV_1 and ΔV_2, in a collinear low velocity collision of vehicles A and B respectively as follows:

$$\Delta V_1 = \frac{1+\epsilon_{AB}}{1+\frac{M_1}{M_2}}(V_2 - V_1) \qquad [11]$$

$$\Delta V_1 = \frac{1+\epsilon_{AB}}{1+\frac{M_2}{M_1}}(V_1 - V_2) \qquad [12]$$