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Energy Basis for Collision Severity

Kenneth L. Campbell
Environmental Activities Staff, General Motors Corp.

ABSTRACT

This paper presents an objective technique for estimating the severity of automobile collisions. The vehicle damage and the dynamic force-deflection characteristics of the vehicle structure are used to estimate the energy absorbed in plastic deformation of the vehicle. This energy can then be expressed as an "equivalent barrier speed" (EBS). The development is limited to frontal damage, although the technique is general and could be extended to side and rear damage.

Data are presented relating residual crush and impact speed for full frontal barrier tests to provide the basis for a simple model of the force-deflection characteristics of the vehicle front structure. EBS is then estimated by integrating this force-deflection characteristic over the deformation of the field vehicle.

The results of this model are compared with test data to indicate the types of damage patterns for which the model appears valid. Calculations are made for damage patterns resembling angle and offset barrier impacts, and the computed EBS is compared with the actual impact speed. For both types of test, the errors seem within normal test variability.

FIELD ACCIDENTS OCCUR over a broad range of collision severities. For a thorough evaluation of any occupant protection system, one would like to determine its effectiveness at various levels of severity. In this approach, the "collision" variables are distinguished from those which describe the vehicle or its occupants. The severity is one aspect of the collision. Other factors in this group include damage location, direction of principal force, object struck, time duration, etc.

In general, no single variable can quantify the total effect of all the collision variables which influence the resulting injury severity. The severity of the collision is only one of these. However, one way to handle these collision variables is to define collision "types" within which the collision severity would be a sufficient basis for comparison (1).* That is, specification of the collision type and severity would completely define the collision, and within any type, the severity measures the contribution of the collision to the resulting injuries. Collision types, then, would be defined in terms of the other collision variables (damage location, direction of principal force, object struck, time duration, etc.).

The primary evidence available for estimating the collision severity is the vehicle damage. For convenience, it is desirable to rate collision severity in a manner which is comparable to an existing test condition like the barrier impact. This is a test in which all vehicles have the same change in velocity and the same energy absorbed per unit weight (assuming equal coefficients of restitution). For a field accident vehicle, the energy absorbed per unit weight can be determined if the force-deflection characteristics of the vehicle and the damage to the vehicle are known. This energy can then be expressed as an "equivalent barrier speed" (EBS). The change in velocity of the field accident vehicle can be computed if the energy absorbed by the struck object and its mass are also known (2).

The implication here is that an "accident file" rather than a "vehicle file" is required if change in velocity is the parameter of interest. However, a vehicle file may be adequate if sufficient research is done to define the relationship of energy absorbed to change in velocity for various types of accidents.

* Numbers in parenthesis designate References at end of paper.

This research would most likely take the form of full scale testing and/or simulation studies.

In any case, the energy absorbed per unit weight of the vehicle is a useful parameter in the estimation of collision severity. For the vehicle file, it may serve as a collision severity measure, while in the accident file, it can also be used to compute changes in velocity. In addition, the use of energy absorbed per unit weight as a basis for "equivalence" with the barrier test, provides a foundation for the development of objective estimation techniques. This paper describes the development of some energy based techniques for estimating equivalent barrier speeds. While the information which follows applies to frontal damage only, the technique is general and could be extended to side and rear damage.

BARRIER TEST DATA

Information on the dynamic force-deflection characteristics of the vehicle front structure is required to relate vehicle deformation to the amount of energy absorbed. This information can be inferred from frontal barrier impact tests. Specifically, residual crush is plotted versus impact speed. These results are shown for '71-'72 full size GM vehicles in Figure 1, and '71-'74 Chevrolet Vegas in Figure 2. All crush measurements were corrected to the "standard weights" shown on the figure by multiplying by the factor:

$$\text{Weight Correction} = \left(\frac{\text{standard weight}}{\text{test weight}}\right)^{1/2} \qquad (1)$$

Weight corrections are described in more detail in the Appendix.

As shown on the Figures, these data can be described by a linear equation of the form:

$$V = b_o + b_1 C \qquad (2)$$

where:

V = impact speed, mph
C = residual crush, in.
b_o = intercept, mph.
b_1 = slope, mph./in.

The intercept, b_o, may be visualized as an impact speed which produces no residual crush. Before any physical significance is attached to this number, it should be pointed out that no data are included at impact speeds below 15 mph (24 km/h). The slope, b_1, was chosen to represent the data as accurately as possible over the range of the data. The resulting value for the intercept, b_o, is simply the extrapolation of this slope to zero crush. Figure 1 indicates that this linear approximation is probably valid over the range of the data presented, 15 - 60 mph (24 - 97 km/h).

Data of this type relating impact speed and residual crush in frontal barrier tests can be summarized by the coefficients of the linear equation, b_o and b_1, and the standard weight at which these coefficients were determined. Table 1 presents these coefficients for the data shown in Figures 1 and 2, as well as the coefficients obtained from similar data on other current GM vehicles. Sufficient information was not available on vehicles not listed in Table 1.

The reader should be cautioned that these coefficients represent an extremely simplified

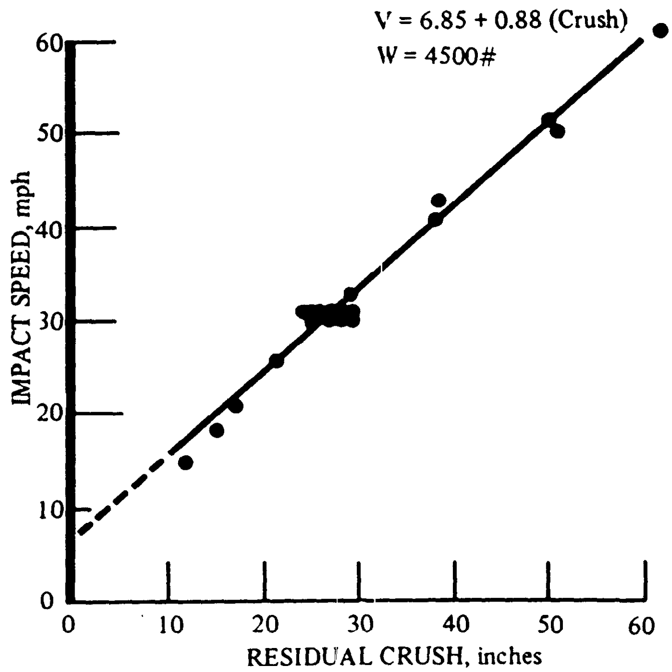


Fig. 1 - Residual crush vs impact speed in full frontal barrier tests for '71-'72 full size GM vehicles

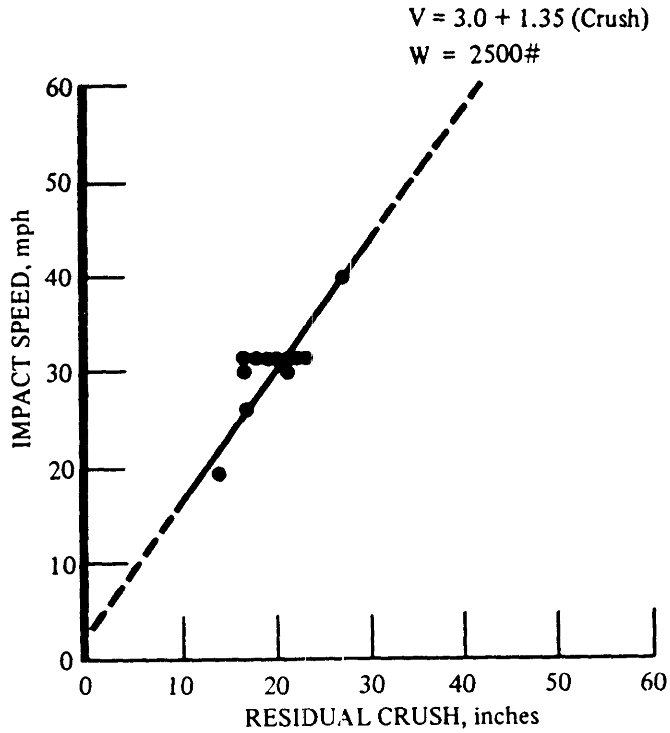


Fig. 2 - Residual crush vs impact speed in full frontal barrier tests for '71-'74 Chevrolet Vegas

Table 1 - Summary - Frontal Barrier Test Data

$$V(mph) = b_o + b_1 C(in.)$$

GM Vehicles	b_o mph	b_1 mph/in.	weight lbs.
71-72 Standard Full Size	6.85	0.88	4500
73-74 Standard Full Size	7.5	0.90	4500
73-74 Intermediate	7.5	0.90	4000
71-74 Compact	3.0	1.35	3400
71-74 Subcompact	3.0	1.35	2500

Vehicle Classification	Examples
Standard Full Size	Chevrolet Impala, Pontiac Catalina, Oldsmobile Delta 88, Buick LeSabre
Intermediate	Chevrolet Chevelle and Monte Carlo, Pontiac LeMans and Grand Prix, Oldsmobile Cutlass, Buick Century
Compact	Chevrolet Nova, Pontiac Ventura, Oldsmobile Omega, Buick Apollo
Subcompact	Chevrolet Vega

Note: The following vehicle classifications are not included in the above table:

Sports	(Chevrolet Corvette)
Compact Specialty	(Chevrolet Camaro, Pontiac Firebird)
Luxury Sedan	(Oldsmobile 98, Buick Electra 225, Cadillac Calais)
Personal Luxury	(Oldsmobile Toronado, Buick Riviera, Cadillac Eldorado)

approximation developed for the purpose of estimating the energy absorbed in plastic deformation of the vehicle front structure. Other applications of this approximation are not likely to be valid. It would also be incorrect to interpret these coefficients as measures of overall vehicle safety.

ESTIMATING EBS

This section presents the development of energy based EBS estimation techniques. The simplest model for the vehicle front structure which will reproduce the linear relationship observed between impact speed and residual crush for the barrier test is a linear force-deflection characteristic. In using this model to estimate the energy absorbed when the damage to the vehicle is not uniform, the assumption is also made that this linear force-deflection characteristic does not vary across the width of the vehicle. That is, the center of the vehicle is no stiffer than the fender area in a frontal impact. Although this assumption may not seem intuitively sound, test data, which will be presented in a later section, indicate it is workable even when as little as 25% of the width of the vehicle is involved. This model also assumes that the damage is uniform vertically. Damage produced by underride or override collisions is not covered. Under these assumptions, the force per unit width, as a function of crush, C, is given by:

$$f = \frac{W}{g w_o} (b_o b_1 + b_1^2 C) \qquad (3)$$

where:

f = force per unit width
W = standard weight
g = gravitational acceleration
b_o, b_1 = coefficients from Table 1 (frontal barrier test)
C = crush
w_o = vehicle width

The energy absorbed (work done) can be computed by integrating this force over the distance crushed, C, to give energy absorbed per unit width, and then integrating over the width of the vehicle. This general expression for computing the energy absorbed may be written as:

$$\text{Energy Absorbed} = \iint f \; dC dw + \frac{W{b_o}^2}{2g} \qquad (4)$$

The last term represents some initial energy which may be absorbed without any residual crush as was implied by the barrier test data presented.

Substituting eq. 3 for f, and integrating with respect to crush, C, yields the following:

Energy Absorbed =

$$\frac{W}{g w_o} \int_o^{w_o} \left(b_o b_1 C + \frac{b_1^2 C^2}{2}\right) dw + \frac{W{b_o}^2}{2g} \qquad (5)$$

Using the energy basis, EBS is defined as a vehicle velocity at which the kinetic energy of the vehicle would equal the energy which was absorbed in plastic deformation. That is:

$$\text{Energy Absorbed} = \frac{1}{2}\frac{W}{g} \; (\text{EBS})^2 \qquad (6)$$

By equating eq. 5 and 6:

$$(\text{EBS})^2 = \frac{1}{w_o} \int_o^{w_o} (2 b_o b_1 C + b_1^2 C^2) \; dw + {b_o}^2 \qquad (7)$$

Various damage patterns can be approximated in terms of the crush, C, as a function of the width, w. The above expression is then evaluated for these situations. For example, a typical damage pattern resulting from angle barrier impacts is shown in Figure 3. For this case:

$$C = C_1 - (C_1 - C_2)\left(\frac{w}{w_o}\right) \qquad (8)$$

Substituting this into eq. 7 and integrating:

$$\text{EBS} = \left[b_o^2 + b_o b_1 (C_1 + C_2) + \frac{b_1^2}{3} (C_1^2 + C_1 C_2 + C_2^2)\right]^{1/2} \qquad (9)$$

EBS is shown graphically as a function of C_1 and C_2 for '71-'72 full size GM vehicles in Figure 4, and '71-'74 Chevrolet Vegas in Figure 5. The appropriate values of b_o and b_1 from Table 1 were substituted into eq. 9 for the two types of vehicles. Also shown on these Figures is a similar damage pattern in which the damage only covers a fraction of the vehicle width, R. A more complete development of these equations is presented in the Appendix.

Figure 6 shows a damage pattern which is intended to resemble the damage produced in an offset barrier impact. In this test only a fraction of the vehicle width contacts the barrier.

For this situation:

$$C = C_1 \text{ when } o \leq w \leq w_1$$

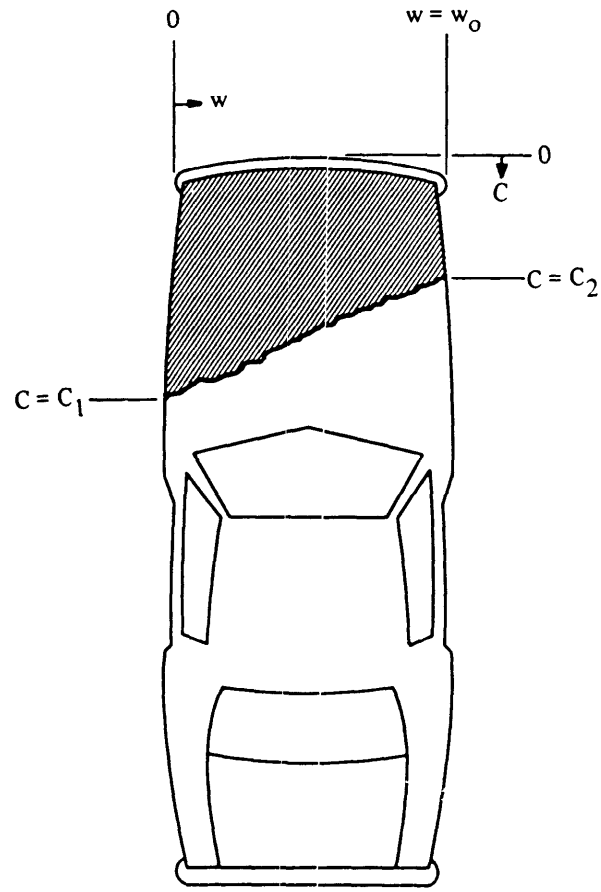
$$C = C_1 - (C_1 - C_2)\left(\frac{w - w_1}{w_o - w_1}\right)^2 \text{ when } w_1 \leq w \leq w_o$$

and

$$\text{EBS} = \left\{b_o^2 + 2 b_o b_1 \left[C_1 R + \frac{(2C_2 + C_1)}{3}(1 - R)\right] + b_1^2 \left[C_1^2 R + \left(\frac{8}{15} C_2^2 + \frac{4}{15} C_1 C_2 + \frac{3}{15} C_1^2\right)(1 - R)\right]\right\}^{1/2} \qquad (10)$$

where

$$R = \frac{w_1}{w_o}$$



$$C = C_1 - (C_1 - C_2)\left(\frac{w}{w_o}\right)$$

Fig. 3 - Damage pattern for angle barrier impacts

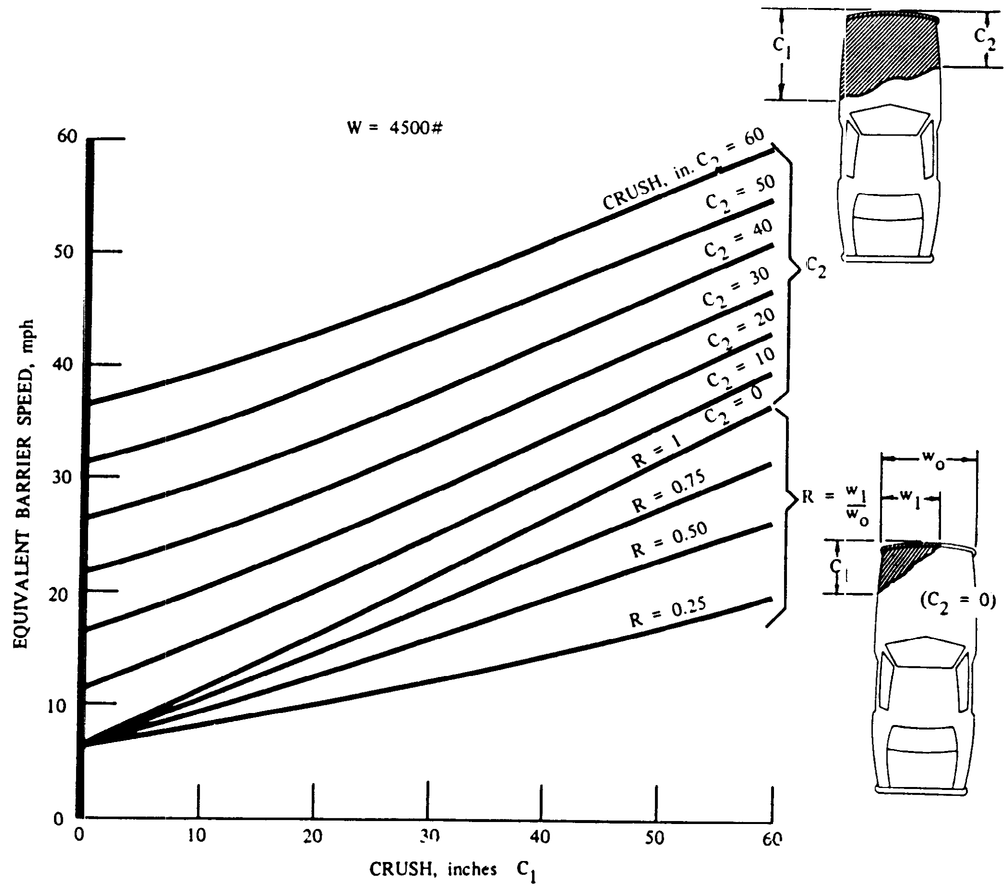


Fig. 4 - EBS vs crush for '71-'72 full size GM vehicles having angle barrier damage patterns

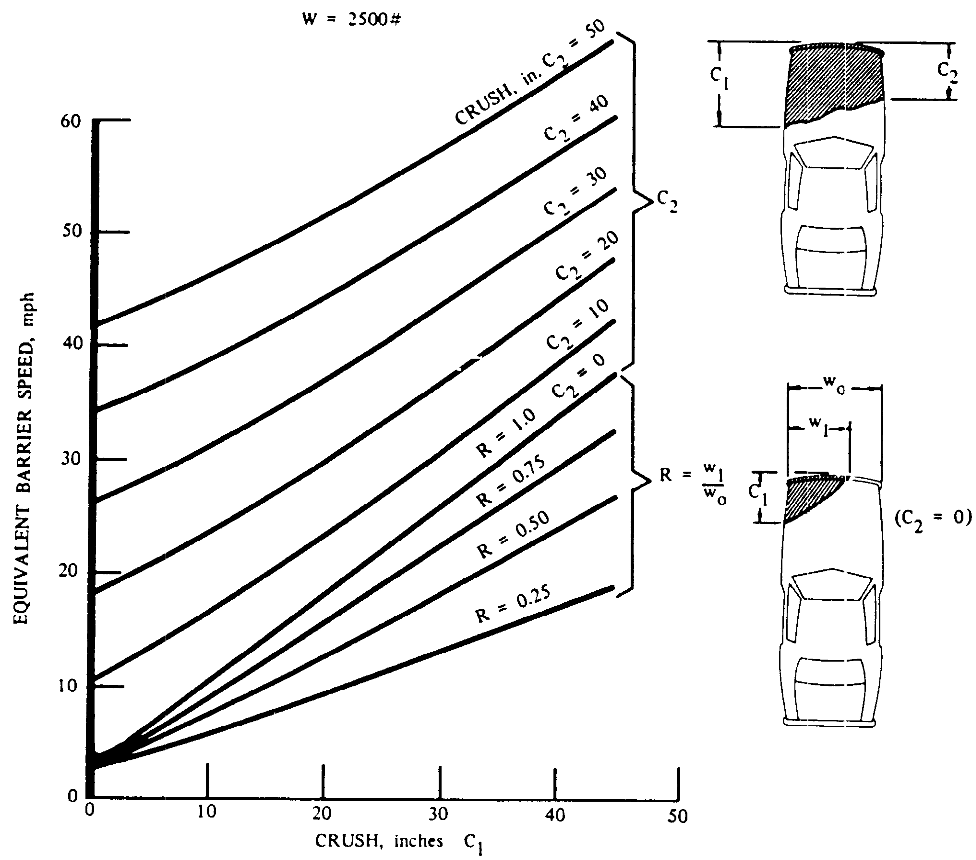
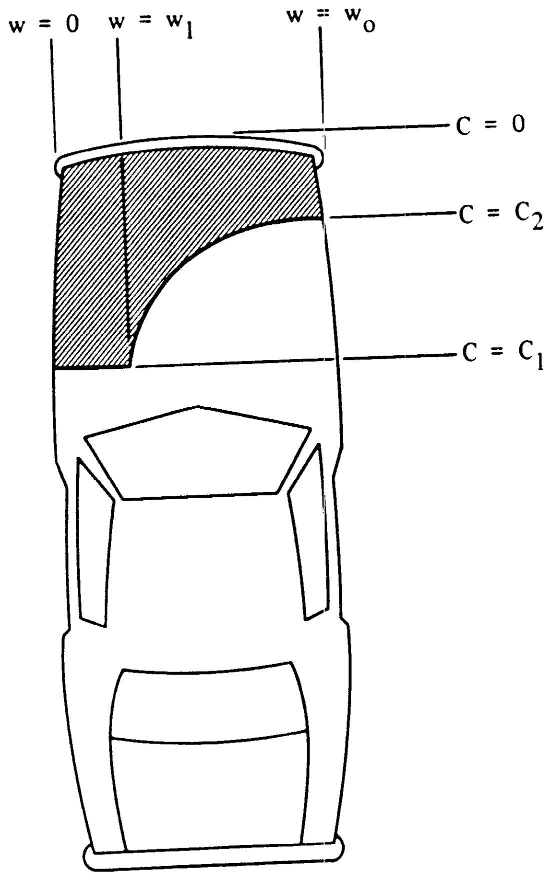


Fig. 5 - EBS vs crush for '71-'74 Chevrolet Vegas having angle barrier damage patterns



$$o \leq w \leq w_1 \qquad C = C_1$$

$$w_1 \leq w \leq w_o \qquad C = C_1 - (C_1 - C_2)\left(\frac{w - w_1}{w_o - w_1}\right)^2$$

Fig. 6 - Damage pattern for offset barrier impacts

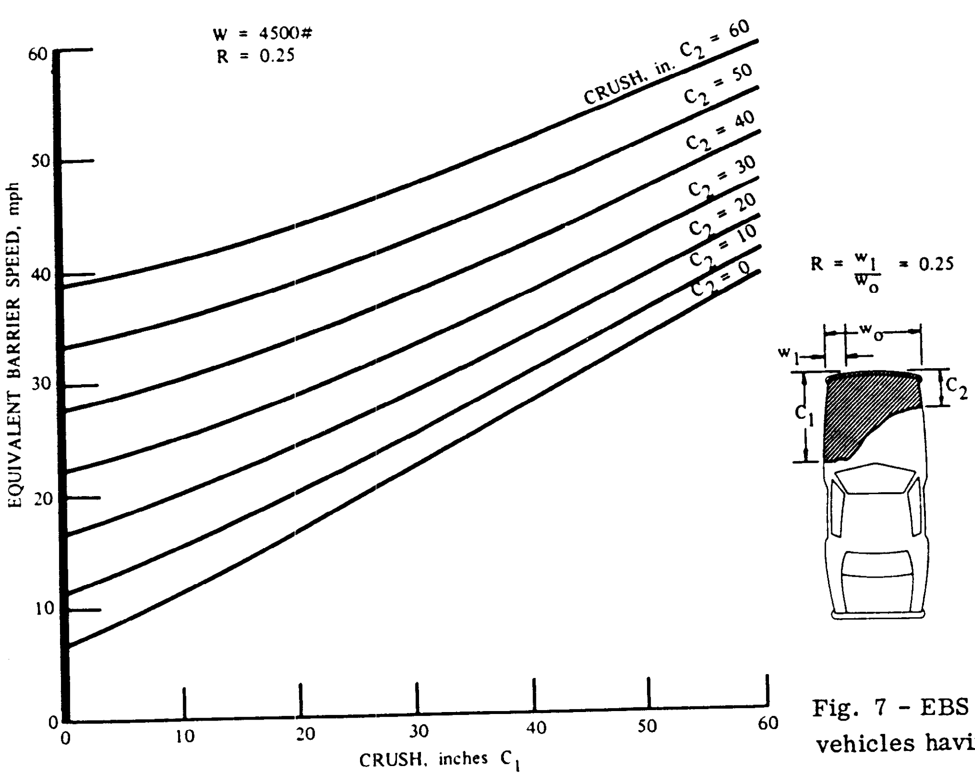


Fig. 7 - EBS vs crush for '71-'72 full size GM vehicles having offset barrier damage patterns

This expression is presented graphically in Figure 7 for the '71-'72 full size GM vehicles and R = .25.

PICTORIAL REPRESENTATION

While the figures based on these equations are quite convenient for common damage patterns, the number of such figures might become unwieldy when one attempts to cover all possible patterns of damage. The use of more sophisticated models for the force-deflection characteristics also complicates application of this approach. An alternative approach is to divide the front of the vehicle into sections and compute the energy each section would absorb. The energy absorbed by any section is given by:

$$\text{Energy Absorbed}\Big]_{w_1,C_1}^{w_2,C_2} = \int_{w_1}^{w_2}\int_{C_1}^{C_2} f \, dC dw \qquad (11)$$

where:

$C_2 - C_1$ = increment in crush

$w_2 - w_1$ = increment in width

By equating eq. 6 and 11:

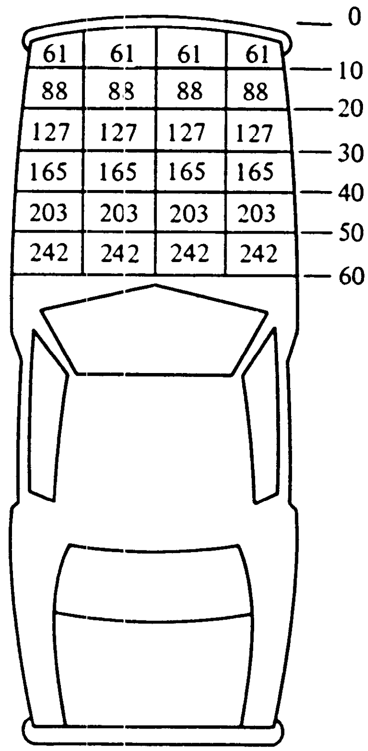
$$(EBS)^2\Big]_{w_1,C_1}^{w_2,C_2} =$$

$$\left[(b_o + b_1C_2)^2 - (b_o + b_1C_1)^2\right]\left(\frac{w_2 - w_1}{w_o}\right) \qquad (12)$$

Figure 8 shows the result when this integration is carried out for the '71-'72 full size GM vehicle using 10 in. (0.25 m) increments in crush and four equal divisions for the width. That is:

$$\frac{w_2 - w_1}{w_o} = \frac{1}{4} \qquad (13)$$

The numbers represent energy absorbed in units of velocity squared. The energy represented by the constant term in eq. 7, b_o^2, has been included in the 0 - 10 in. (0.25 m) crush region. To use this figure, the damage is simply sketched on the vehicle pictured. Total energy absorbed is determined by adding the numbers in the "crushed" squares. Partial squares are allotted in proportion to area. Since these numbers represent energy absorbed in units of velocity squared, the square root of their sum is the EBS. Illustrations of the techniques developed are given in the following section.



CRUSH (inches) vs $(EBS)^2$
71-72 FULL SIZE CHEVROLET
12 O'CLOCK DIRECTION OF FORCE
>25% CONTACT
W = 4500#

Fig. 8 - Pictorial representation of crush vs $(EBS)^2$ for '71-'72 full size Chevrolet

MODEL EVALUATION

The linear force-deflection model is based upon data from full frontal barrier tests. Before this model is used to estimate the energy absorbed by field accident vehicles, some verification is needed to insure that this model is reasonably valid when the damage pattern is different from that produced in the barrier test. Data from some angle and offset barrier tests are presented as a partial validation of the model. The damage patterns assumed in Figures 4, 5, and 7, correspond to the damage produced in these tests. Because they are barrier tests, essentially all the impact energy is absorbed by the vehicle so that the estimated EBS should equal the impact speed of the test.

The comparison was made by using the appropriate expression, eq. 9 or 10, to compute the EBS based on the measured crush and the coefficients from Table 1. Weight corrections were then applied as described in the Appendix. The results are presented in Tables 2 and 3. The error was computed by subtracting the actual impact speed from the estimated EBS.

Table 2 - Comparison of Angle Barrier Test Data With Results of Linear Force-Deflection Model

Test No.	Impact Speed	Test Weight	Left Crush	Right Crush	Estimated Speed	Error
			Chevrolet Vega			
1	30.6 mph	2390 lbs.	35.0 in.	6.5 in.	33.7 mph	+3.1 mph
			Standard Full Size GM Vehicles			
2	18.3	4715	0	25.0	19.4	+1.1
3	21.6	4873	29.0	0	21.0	-0.6
4	21.7	5032	38.0	2.0	25.7	+4.0
5	21.5	4730	1.0	28.0	21.2	-0.3
6	25.4	4536	39.5	0.5	27.3	+1.9
7	30.6	4820	37.0	2.0	25.7	-4.9
8	30.8	4808	43.0	5.5	29.9	-0.9
9	30.8	4505	40.0	0	27.5	-3.3
10	29.5	4573	36.0	0	25.3	-4.2
11	30.7	4561	3.0	49.5	33.2	+2.5
12	30.4	4835	4.0	40.0	27.8	-2.6

Average Error = -0.35 mph (0.6 km/h)

Table 3 - Comparison of Offset Barrier Test Data With Results of Linear Force-Deflection Model

Test No.	Impact Speed	Test Weight	Left Crush	Right Crush	Estimated Speed	Error
			Compact R = 0.25			
1	29.4 mph	3611 lbs.	29.0 in.	1.5 in.	26.8 mph	-2.6 mph
2	29.1	3601	31.0	0	28.1	-1.0
3	30.7	3596	34.0	3.0	31.4	+0.7
			Standard Full Size GM Vehicles R = 0.25			
4	30.0	4233	42.2	6.2	31.6	+1.6
5	30.6	4192	40.0	0	28.9	-1.7
6	30.6	4333	41.0	3.0	29.7	-0.9
7	30.7	4395	43.5	6.5	31.8	+1.1
			Chevrolet Vega R = 0.25			
8	30.6	2543	35.0	0	32.0	+1.4
9	30.0	2388	29.0	3.0	28.8	-1.2
10	30.5	2373	34.0	4.5	33.7	+3.2
			Standard Full Size GM Vehicle R = 0.75			
11	30.9	4943	18.0	30.0	30.2	-0.7

Average Error = -0.01 mph (0.016 km/h)

For the angle barrier tests, the average difference between the estimated and actual impact speed was -0.4 mph (0.6 km/h), while for the offset barrier tests this difference was essentially zero. The range of the errors is ± 4 mph (± 6.4 km/h) and ± 3 mph (± 4.8 km/h), respectively. These variations are normally observed in barrier test data as is illustrated by the ± 3 mph (± 4.8 km/h) scatter shown in Figures 1 and 2 at 30 mph (48 km/h). Since the

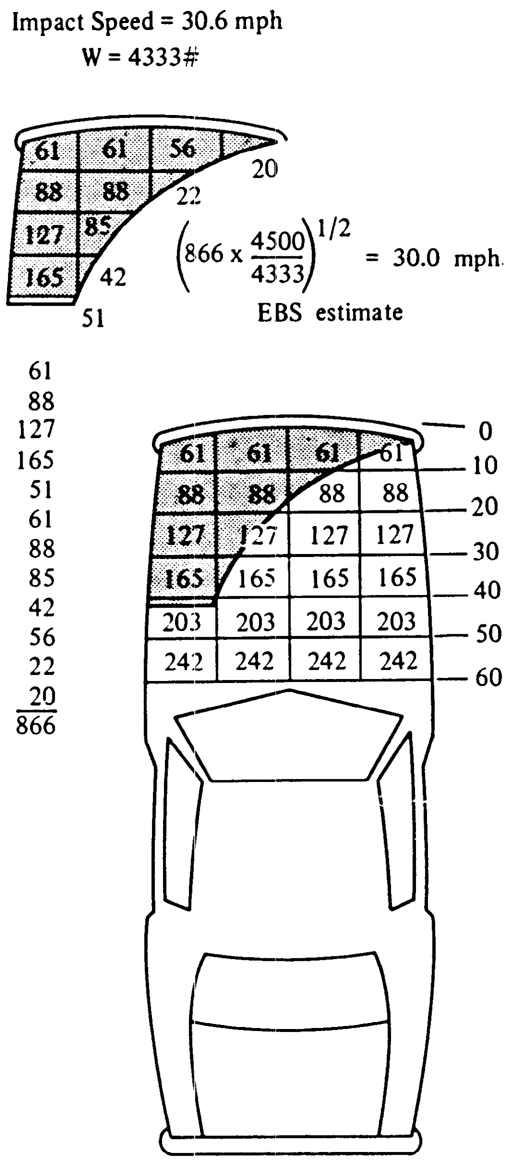


Fig. 9 - Application of pictorial approach to 30 mph offset barrier impact

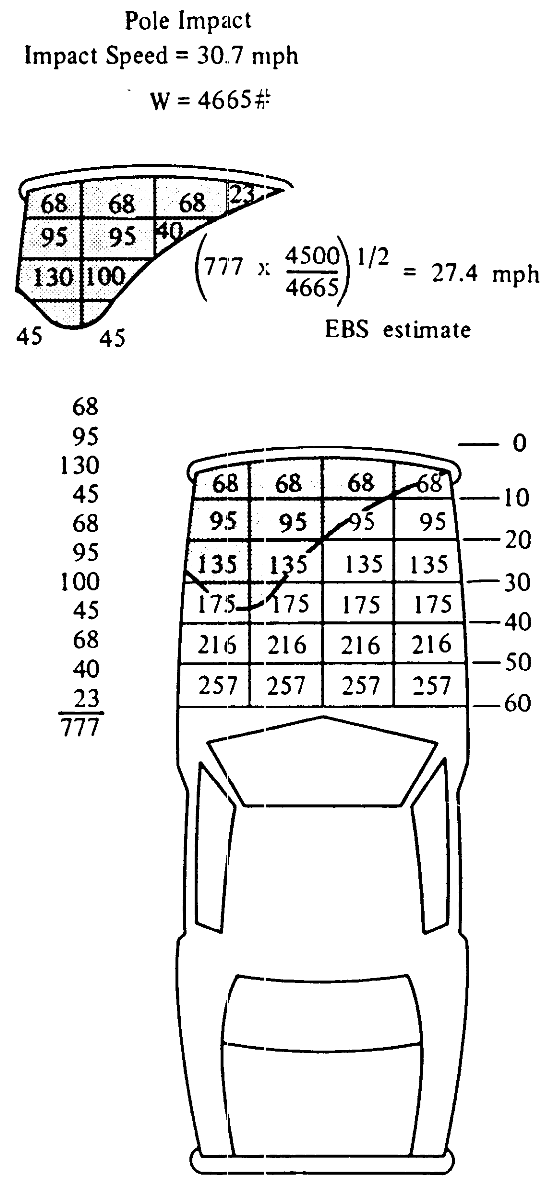


Fig. 10 - Application of pictorial approach to 30 mph offset pole impact

errors have a nearly zero mean, there is no evidence of any consistent errors for these two applications.

Figures 9 and 10 show applications of the pictorial approach. The damage sketched in Figure 9 corresponds to test number 6 on Table 3. After applying the weight correction, the resulting estimate is 30.0 mph (48.3 km/h), using the pictorial approach. Equation 10 produced an estimate of 29.7 mph (47.8 km/h). Figure 10 shows the damage resulting from a 30.7 mph (49.4 km/h) frontal impact into a 14 in. (0.36 m) pole. As shown on the Figure, the estimated EBS is 27.4 mph (44.1 km/h) after correction for weight.

The results presented in this section indicate the validity of the model and the accuracy of the method. The model appears to be adequate for the patterns of damage produced by the angle and offset barrier tests. In applying these techniques to field accident vehicles, the damage pattern should be considered rather than the object struck. The validation for the angle barrier is important because the pattern of damage produced by this test occurs frequently in field accidents.

The damage pattern produced by the offset barrier test probably does not occur as frequently, but these results are also of interest because they represent a fairly severe test of the assumption that the force-deflection characteristic does not vary across the width of the vehicle. From these results, the conclusion can be drawn that the model appears valid as long as at least 25% of the vehicle width is contacted. In all of these tests, the direction of force was longitudinal to the vehicle. Different force-deflection characteristics may be exhibited if the direction of force were different.

These tests also give a good indication of the accuracy to be expected. As illustrated by the scatter at 30 mph (48 km/h) in Figures 1 and 2, errors ranging ± 3 mph would occur if impact speed were estimated on the basis of crush for the frontal barrier test. From Tables 2 and 3 the range of the errors is ± 4 mph (± 6.4 km/h) and ± 3 mph (± 4.8 km/h), respectively. In field applications, the crush measurements are more difficult to obtain and the damage more irregular. In addition, some error is always introduced by modeling the structural characteristics. Based on these considerations, a range of at least ± 5 mph (± 8 km/h) should be attached to any EBS estimate. Since most of the data used here was at 30 mph (48 km/h), accuracy could also be expected to decrease as one approaches the limits of the 15 - 60 mph (24 - 97 km/h) range given.

SUMMARY

These results by no means represent an exhaustive validation of the model used. The need for more sophisticated force-deflection information may arise if validation is attempted for additional patterns of damage (underride or override). However, the validity of the basic approach, in which the energy absorbed in plastic deformation of the vehicle is used as the basis for estimating EBS, is not affected by the validity of this simple model. With this definition, EBS can be determined as accurately as desired; the limiting factor is the effort expended in determining the vehicle force-deflection characteristic and recording the field vehicle deformation.

"Equivalence" with the barrier impact, or any other field accident, is based only on energy absorbed. EBS may be used as a measure of collision severity. However, other collision variables (damage location, direction of principal force, object struck, time duration, etc.) also influence the <u>potential for injury</u> of the collision. All frontal damage collisions at a given EBS do not have the same potential for injury. For example, nearly identical frontal damage and collision severity (EBS) may result from impacts with a fixed rigid barrier and a roadside "gore" barrier. The potential for injury in these two impacts is very different based on the greatly increased stopping distance and time duration of the impact with the yielding gore area. In this case, the rate at which the energy is absorbed significantly affects the injury potential of the collision. One approach may be to group struck objects by their stiffness. EBS would be an appropriate measure of collision injury potential within groups of frontal damage vehicles which struck objects of comparable stiffness. In other words, the stiffness of the struck object would be one of the variables used to define the collision types referenced earlier.

A thorough understanding of the collision and injury mechanisms is necessary to insure that all appropriate collision variables are included in the definition of collision types. Once these collision types have been adequately defined, collision severity may be used within each type as a measure of collision injury potential. At this point, one may begin to use field accident data to evaluate the effectiveness of restraint systems. The restraint system is a major factor which determines the actual injury severity resulting from any given level of collision severity. Other occupant-related variables also influence the relationship between collision severity and injury severity (2). These factors must be considered when one attempts to evaluate the effectiveness of any occupant protection system using field accident data.

The real value of defining EBS on the basis of energy absorbed is the foundation this approach provides for the development of <u>objective</u> estimation techniques. Widespread acceptance of this definition would provide a much-needed "common ground" for the analysis of field accident data.

REFERENCES

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2. J. F. Marquardt, "Vehicle and Occupant Factors That Determine Occupant Injury." Paper 740303, presented at S.A.E. Automotive Engineering Congress, Detroit, Michigan, February 1974.

APPENDIX

DERIVATIONS - In general, the energy absorbed (work done) in plastic deformation of a structure can be obtained by integrating the local force per unit area over the volume of deformation. If the damage is uniform over the vertical dimension, then this integration can be eliminated, leaving only the integrations with respect to crush and width as given by:

$$\text{Energy Absorbed} = \iint f \, dCdw + \text{Constant} \quad \text{(A-1)}$$

The constant is needed to include some initial energy which is absorbed with no residual crush as implied by the non-zero intercept on Figures 1 and 2.

A linear equation of the form

$$V = b_o + b_1 C \quad \text{(A-2)}$$

has been selected to describe the relationship of residual crush, C, and impact speed, V, in the full frontal barrier test. A linear model of the form

$$f = a_o + a_1 C \quad \text{(A-3)}$$

is desired to describe the force per unit width of the front structure as a function of crush. Eq. A-1 can be applied to the full frontal barrier test to relate the coefficients of eq. A-3, a_o and a_1, to the known coefficients of eq. A-2, b_o and b_1.

In the barrier impact all the kinetic energy of the vehicle at impact is absorbed, and equation A-1 may be written as:

$$\frac{1}{2}\frac{W}{g}V^2 = \int_0^{w_o}\int_0^{C}(a_o + a_1 C)\, dCdw + \text{Constant} \quad \text{(A-4)}$$

Eq. A-3 has been substituted for f. Substituting eq. A-2 for V and integrating.

$$\frac{1}{2}\frac{W}{g}(b_o + b_1 C)^2 = a_o C w_o + a_1 \frac{C^2}{2} w_o + \text{Constant} \quad \text{(A-5)}$$

Solving for a_o, a_1, and the constant:

$$\text{Constant} = \frac{1}{2}\frac{W}{g}b_o^2$$

$$a_o = \frac{W\, b_o b_1}{g\, w_o}$$

$$a_1 = \frac{W\, b_1^2}{g\, w_o}$$

Therefore, the force per unit width can be expressed in terms of b_o and b_1 as:

$$f = \frac{W}{g w_o}(b_o b_1 + b_1^2 C) \quad \text{(A-6)}$$

When the damage is not uniform like that produced in the barrier test, the energy absorbed may be estimated by:

$$\text{Energy Absorbed} = \frac{W}{g w_o}\iint (b_o b_1 + b_1^2 C)\, dCdw + \frac{W}{2g} b_o^2 \quad \text{(A-7)}$$

Integrating with respect to C:

$$\text{Energy Absorbed} = \frac{W}{g w_o}\int \left(b_o b_1 C + \frac{b_1^2 C^2}{2}\right) dw + \frac{W}{2g} b_o^2 \quad \text{(A-8)}$$

If EBS is defined by:

$$\text{Energy Absorbed} = \frac{1}{2}\frac{W}{g}(\text{EBS})^2 \quad \text{(A-9)}$$

Then

$$\frac{(\text{EBS})^2}{2} = \frac{1}{w_o}\int \left(b_o b_1 C + \frac{b_1^2 C^2}{2}\right) dw + \frac{b_o^2}{2} \quad \text{(A-10)}$$

If the damage pattern can be expressed in terms of the crush, C, as a function of the width, then this integration can be completed. For example, the damage pattern shown in Figure 3 can be described by

$$C = C_1 - (C_1 - C_2)\left(\frac{w}{w_o}\right) \quad \text{(A-11)}$$

Rewriting eq. A-10

$$\frac{(\text{EBS})^2}{2} = \frac{b_o b_1}{w_o}\int_0^{w_o} C dw + \frac{b_1^2}{2 w_o}\int_0^{w_o} C^2 dw + \frac{b_o^2}{2} \quad \text{(A-12)}$$

Substituting eq. A-11 and carrying out the first integration.

$$\frac{1}{w_o}\int_0^{w_o} C dw = \frac{1}{w_o}\int_0^{w_o}\left[C_1 - (C_1 - C_2)\frac{w}{w_o}\right] dw$$

$$= \frac{C_1}{w_o}\int_0^{w_o} dw - \frac{(C_1 - C_2)}{w_o^2}\int_0^{w_o} w dw$$

$$= \frac{C_1}{w_o} w_o - \frac{(C_1 - C_2)}{w_o^2} \frac{w_o^2}{2}$$

$$= C_1 - \frac{(C_1 - C_2)}{2}$$

$$= \frac{(C_1 + C_2)}{2} \qquad \text{(A-13)}$$

Substituting eq. A-11 and carrying out the second integration.

$$\frac{1}{w_o}\int_o^{w_o} C^2 dw = \frac{1}{w_o}\int_o^{w_o}\left[C_1 - (C_1 - C_2)\frac{w}{w_o}\right]^2 dw$$

$$= \frac{C_1^2}{w_o}\int_o^{w_o} dw - 2\frac{C_1(C_1 - C_2)}{w_o^2}\int_o^{w_o} wdw$$

$$+ \frac{(C_1 - C_2)^2}{w_o^3}\int_o^{w_o} w^2 dw$$

$$= C_1^2 - C_1(C_1 - C_2) + \frac{(C_1 - C_2)^2}{3}$$

$$= \frac{C_1^2 + C_1C_2 + C_2^2}{3} \qquad \text{(A-14)}$$

Substituting A-13 and A-14 into A-12

$$\frac{(EBS)^2}{2} = b_o b_1 \frac{(C_1 + C_2)}{2} + \frac{b_1^2}{2}\frac{(C_1^2 + C_1C_2 + C_2^2)}{3} + \frac{b_o^2}{2}$$

Rearranging and taking the square root:

$$EBS = \left[b_o^2 + b_o b_1 (C_1 + C_2) + \frac{b_1^2}{3}(C_1^2 + C_1C_2 + C_2^2)\right]^{1/2} \qquad \text{(A-15)}$$

A similar damage pattern is shown in Figures 4 and 5. Here the damage only covers a fraction of the vehicle width, R. By changing the limit of integration to w_1 in eq. A-12 and setting C_2=0 and replacing w_o with w_1 in eq. A-11, it may be seen that the EBS for this damage pattern is given by:

$$EBS = \left[b_o^2 + R(b_o b_1 C_1 + \frac{b_1^2}{3} C_1^2)\right]^{1/2} \qquad \text{(A-16)}$$

where: w_1 = width of vehicle damaged

$$R = \frac{w_1}{w_o}$$

Figure 7 shows the damage pattern chosen to represent the offset barrier impact. The following describes the crush as a function of the width:

$$o \le w \le w_1 \qquad C = C_1 \qquad \text{(A-17)}$$

$$w_1 \le w \le w_o \qquad C = C_1 - (C_1 - C_2)\left(\frac{w - w_1}{w_o - w_1}\right)^2 \qquad \text{(A-18)}$$

In this case the two integrals of eq. A-12 must each be broken into two parts:

$$\frac{1}{w_o}\int_o^{w_o} Cdw = \frac{1}{w_o}\int_o^{w_1} Cdw + \frac{1}{w_o}\int_{w_1}^{w_o} Cdw \qquad \text{(A-19)}$$

and

$$\frac{1}{w_o}\int_o^{w_o} C^2 dw = \frac{1}{w_o}\int_o^{w_1} C^2 dw + \frac{1}{w_o}\int_{w_1}^{w_o} C^2 dw \qquad \text{(A-20)}$$

Substituting eq. A-17 into the integrals over the range of $o \le w \le w_1$ and eq. A-18 into the integrals over the range $w_1 \le w \le w_o$ and integrating:

$$\frac{1}{w_o}\int_o^{w_o} Cdw = \frac{(2\,C_2 + C_1)}{3}(1-R) + C_1 R \qquad \text{(A-21)}$$

$$\frac{1}{w_o}\int_o^{w_o} C^2 dw = \left(\frac{8}{15}C_2^2 + \frac{4}{15}C_1C_2 + \frac{3}{15}C_1^2\right)(1-R)$$

$$+ C_1^2 R \qquad \text{(A-22)}$$

where:

$$R = \frac{w_1}{w_o}$$

Substituting into eq. A-12:

$$\frac{(EBS)^2}{2} = b_o b_1\left[\frac{(2C_2 + C_1)}{3}(1 - R) + C_1 R\right]$$

$$+ \frac{b_1^2}{2}\left[\left(\frac{8}{15}C_2^2 + \frac{4}{15}C_1C_2 + \frac{3}{15}C_1^2\right)(1 - R) + C_1^2 R\right] + \frac{b_o^2}{2}$$

Rearranging:

$$EBS = \left\{b_o^2 + 2b_o b_1\left[C_1 R + \frac{(2C_2 + C_1)}{3}(1 - R)\right] + b_1^2\left[C_1^2 R + \left(\frac{8}{15}C_2^2 + \frac{4}{15}C_1C_2 + \frac{3}{15}C_1^2\right)(1 - R)\right]\right\}^{1/2} \qquad \text{(A-23)}$$

WEIGHT CORRECTIONS - In deriving eq. A-10, the weight of the field vehicle was assumed to equal the standard weight at which the coefficients, b_o and b_1, were determined so that W could be cancelled on each side of the equation. If these weights are not equal, eq. A-10 must be written as:

$$W_f \frac{(EBS)^2}{2} = \frac{W_s}{w_o} \int (b_o b_1 C + \frac{b_1^2 C^2}{2}) dw + \frac{b_o^2}{2} \quad \text{(A-24)}$$

where:

W_f = weight of field vehicle, lbs.
w_s = standard weight, lbs.

Rearranging and taking the square root:

$$EBS = \left(\frac{W_s}{W_f}\right)^{\frac{1}{2}} \left[\frac{1}{w_o} \int (2b_o b_1 C + b_1^2 C^2) dw + b_o^2\right]^{\frac{1}{2}} \quad \text{(A-25)}$$

By comparison of eq. A-10 and A-25, the term,

$$\left(\frac{W_s}{W_f}\right)^{\frac{1}{2}}$$

can be thought of as a correction factor which multiplies the results of eq. A-10 when $W_s \neq W_f$.

The same weight correction was applied to the crush measurements shown in Figures 1 and 2, and used to determine the coefficients, b_o and b_1, in Table 1 at the desired standard weight. This application is only exact if b_o=0. However, the error involved can be shown to be quite small for typical values of b_o.

First, eq. A-24 is integrated for the barrier test (C constant over the width, w, and V = Impact Speed = EBS) to give:

$$\frac{W_f}{2} V^2 = \frac{W_s}{2} (2b_o b_1 C_f + b_1^2 C_f^2) + \frac{W_s}{2} b_o^2$$

The subscript, f, now refers to a barrier test vehicle at a weight different from W_s. Solving for C_f:

$$C_f = \frac{1}{b_1} \left[b_o + \left(\frac{W_f}{W_s}\right)^{\frac{1}{2}} V\right] \quad \text{(A-26)}$$

The desired correction is $\frac{C_s}{C_f}$ so that the multiplication of C_f by this factor will yield the crush which would have been produced at the same impact speed and the standard weight, W_s. Forming this ratio using eq. A-26:

$$\frac{C_s}{C_f} = \frac{\frac{1}{b_1}\left[b_o + \left(\frac{W_s}{W_s}\right)^{\frac{1}{2}} V\right]}{\frac{1}{b_1}\left[b_o + \left(\frac{W_f}{W_s}\right)^{\frac{1}{2}} V\right]}$$

or:

$$\frac{C_s}{C_f} = \frac{b_o + V}{b_o + \left(\frac{W_f}{W_s}\right)^{\frac{1}{2}} V} \quad \text{(A-27)}$$

Note that if $b_o = 0$, $\frac{C_s}{C_f} = \left(\frac{W_s}{W_f}\right)^{\frac{1}{2}}$ (A-28)

The error produced by using this term instead of eq. A-27 is ~1% if W_s = 4500 lbs., W_f = 5000 lbs., V = 30 mph and b_o = 7 mph.

A correction for slight variations in test speed may also be deduced from eq. A-27 as:

$$\frac{C_s}{C_f} = \frac{b_o + V_s}{b_o + V_f} \quad \text{(A-29)}$$

if $b_o = 0$

$$\frac{C_s}{C_f} = \frac{V_s}{V_f} \quad \text{(A-30)}$$

The error produced by using the velocity ratio (A-30) instead of A-29 is ~6% if b_o = 7, V_s = 30 mph, and V_f = 31 mph. These two corrections, eq. A-28 and A-30, allow crush measurements to full frontal barrier tests to be corrected to a standard weight and impact speed. The advantage in using these over eq. A-27 and A-29, of course, is that b_o and b_1 are not known before the data are analyzed.